Effectiveness and Relay Efficiency of Opportunistic Multipoint Relaying on Cooperative ARQ

Sau-Hsuan Wu, Hsin-Li Chiu, and Jin-Hao Li

***Abstract*—This paper studies the effectiveness and relay efficiency of using opportunistic multipoint relaying and distributed space-time coding (ODSTC) for cooperative Automatic Repeat reQuest (ARQ). According to the complexities for diversity exploration in retransmissions, three types of ARQ protocols are studied herein, allowing us to examine the effectiveness of the protocols and the efficiencies of using ODSTC relaying for ARQ. The efficiency is studied from the viewpoint of the signal quality enhancement per active relay, while the effectiveness is investigated from the perspectives of diversity and throughput improvements with the different protocols. The results show that both are closely related to the link qualities of the forwarding channels. When the link quality between the source and relays is considerably higher than the quality between the relays and the destination, a simple protocol that opportunistically chooses two active relays for ODSTC (ODSTC2) at the beginning of retransmissions is good enough to yield a significant throughput improvement. Otherwise, when the performance is limited by the number of successfully decoding relays, then having relays of unsuccessful decoding to overhear the signals sent by active relays plays a crucial role in resolving the limitation. By having the new successfully decoding relays to participate in the subsequent retransmissions, both diversity and throughput can be significantly improved. According to our analysis and simulation studies, an appropriate protocol with ODSTC2 in general can provide effective throughput improvements in two times retransmissions.**

***Index Terms*—Distributed space-time coding (DSTC), diversity analysis, opportunistic multipoint relaying (OMPR), relay-assisted Automatic Repeat reQuest (ARQ).**

I. Introduction

Cooperative communications have emerged as a new paradigm for wireless communications. A host of cooperative schemes have been introduced to enhance the transmission capacity and/or reliability since the work of [1]–[4], either through user cooperations or via signal relaying (see [5]–[12] and the references therein). In view of the cost advantages of relay stations and the performance enhancement that can be brought about with signal relaying, cooperative relaying has been incorporated in international standards such as the Third-Generation Partnership Project (3GPP) Long-Term Evolution Advanced (LTE-A) Release [13].

To exploit the spatial diversity offered by multiple relays, distributed space-time coding (DSTC) and beamforming schemes have been widely studied and reported in literatures (see e.g. [7], [12], and [14], among others), following the notions of decode-and-forward (DF) or amplify-and-forward (AF) relaying introduced in [2] and [3]. However, to fully exploit the rich diversities offered by multipoint relaying (MPR), the design and implementation of DSTC remain to be a challenging task in practice [15] when considering the changing numbers of relays that can successfully decode the data. In view of the complexities of using DSTC, an opportunistic relaying (OR) scheme is proposed in [10], where only the relay that possesses the best link quality to the destination forwards the signal. Though simple, the OR scheme still enjoys the full diversity offered by the entire set of relays, and is shown to be optimal when subject to a total power budget for the entire set of relays. Extending the idea of OR, DSTC is reexamined in [16] for opportunistic MPR (OMPR), followed by a more general study on OMPR in orthogonal channels in [17].

In contrast to the rich diversities offered by cooperative relaying, the multiplexing gain of relaying is typically limited in the case of half-duplex relaying (HDR) [3]. To cope with this limitation, a dynamic DF relaying scheme is proposed in [5], whose diversity–multiplexing tradeoff (DMT) is shown to achieve the DMT upper bound of multiple-input and single-output (MISO) channels [3] when the multiplexing gain is less than 0.5. If relays have the channel state information prior to transmissions, Yuksel and Erkip [9] show that the MISO DMT upper bound can be achieved with the compress-and-forward HDR [4].

More recently, full-duplex relaying (FDR) is also introduced in [18] to improve the DMT. Alternative to FDR and the aforementioned physical-layer methods, cross-layer approaches such as Automatic-Repeat-reQuest (ARQ) can be combined with cooperative relaying to improve the DMT of HDR as well [6], [19]–[21]. Among them, Stanojev *et al.*[19] apply DSTC for relay-assisted ARQ, and show that the effective throughput can be improved either with a typical ARQ or with a hybrid ARQ (HARQ) of the type of chase combining. Exploiting the extra degrees of freedom offered by relay-assisted ARQ, Azarian *et al.*[6], [20] demonstrate that both the diversity and the multiplexing gains can be enhanced by using incremental redundancy for HARQ. Besides, motivated by the simplicity of OR, relay selection schemes are further studied in [22], and the references

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therein, for relay-assisted ARQ to exploit the spatial and temporal diversities with opportunistic AF relaying [10].

The aforementioned results suggest that making use of relays for signal retransmissions can be a promising technology for system throughput or coverage enhancement. Continuing with this idea, relay selection schemes are studied in [23] for ARQ with DF relaying. Performance of two important relay selection schemes are carefully examined therein: the reactive scheme that reselects a relay according to the rule of OR whenever a retransmission is needed, and the adaptive scheme that does relay reselection only when the channels to or from the current active relay are too weak to successfully receive a packet from the source or to deliver a packet to the destination. The results show that the diversity gain offered by OR is often compromised by the overheads from unnecessary relay reselections in the reactive scheme when the channel can still support successful relaying. The adaptive scheme avoids this situation by doing relay reselection only when needed. Even in fast fading channels, the diversity gain by OR can be offset by the overheads from frequent relay reselections, leading to no relay reselection as the best strategy in such channels.

The above results reveal that the effectiveness of relay-assisted ARQ depends on various system design factors, such as the number of participating relays, the distributed coding schemes, the channel conditions, and the complexity for relays coordination. This motivates us to reinvestigate the MPR-based ARQs from a system perspective, aiming to study the effectiveness and efficiency of OMPR on throughput enhancement. To this end, we limit our research scope to the diversity aspect of OMPR only, and focus on the applications of opportunistic DSTC (ODSTC) to the selective-repeat ARQ method.

We begin with a quantitative analysis to evaluate the loss of the signal-to-noise ratio (SNR) gain in capacity outage probability when using only partial available relays rather than all of them for ODSTC. The results not only allow us to compare the performance of the ODSTC-based relaying to that of the regular DSTC-based relaying, but also help justify the efficiency of ODSTC relaying against the simple OR scheme. Based on the results, we further study three types of ARQ protocols, which require different degrees of coordinations to exploit the spatial and temporal diversities of using ODSTC in retransmissions. From complexity low to high, the three protocols, denoted by Type-A, -B, and -C in the sequel, add one more function of relay selection, reselection, and overhearing into their ARQ schemes in order to exploit more degrees of diversities in retransmissions. Despite the overhearing function of the Type-C protocol, which requires relays of unsuccessful decoding to continue overhearing signals sent from active relays, the three protocols are in fact the same for the first round of retransmissions, allowing us to evaluate the effectiveness of the additional procedures for diversity enhancement.

In addition to the above comparative studies on relays efficiencies, we also examine the influence of the link qualities of relay channels on the performance of the three types of protocols. Analysis shows that the effectiveness of these protocols is highly dependent on the relative rate settings and channel qualities among the source, relays, and the destination. When the ratio of the link quality between the source and the relays (S-R) to the link quality between the relays and the destination (R-D) is high enough, the relay channel degenerates, at high SNR, to a MISO channel with antenna selection. Under this channel condition, the Type-B protocol can achieve the same diversity of the Type-C protocol. On the contrary, when the R-D to S-R link ratio is high enough, then the relay channel resembles a single-input and multiple-output (SIMO) channel. The destination in this case can decode the data as long as any one of the relays is able to do so. As a result, the three types of protocols perform exactly the same under this channel condition. Other than these two special operating conditions, the diversities of the capacity outage probabilities of ARQs are typically limited by the cardinality $\mathcal{D}$ of the set $\mathcal{S}_D$ of relays that successfully decode the data. Only the Type-C protocol can resolve this relay shortage problem and achieve the full diversity of relay-assisted ARQ by having relays $\notin \mathcal{S}_D$ overhear the signals sent by the active relays $\in \mathcal{S}_D$, and then rejoin ODSTC relaying once $\in \mathcal{S}_D$ in the subsequent ARQs.

In addition to diversity analysis, simulation studies also show that the ratio of the S-R to R-D link qualities has a comparative impact on the system throughput and protocol effectiveness. When the ratio is close to or slightly less than one such that the performance is mainly limited by the available relays in $\mathcal{S}_D$, then using only two active relays for ODSTC (ODSTC2) in the Type-C protocol can effectively resolve this relay shortage problem and offer significant throughput enhancement. On the contrary, when the ratio becomes considerably large such that the performance is mainly limited by the R-D link quality, either the Type-A with ODSTC2 or the Type-B with OR provides a satisfactory throughput. In spite of the results presented herein, we note that ODSTC-based ARQ may have a more profound influence on the system throughput from a cross-layer point of view. Some results on cross-layer analysis for relay-assisted ARQ can be found in [24]–[26].

The paper is organized as follows. According to the system setting in Section II, the capacity outage probability of ODSTC-based relaying is analyzed in Section III. Based on the results, three types of ODSTC-based ARQ protocols and their capacity outage probabilities are studied in Section IV, followed by the diversity and the relay efficiency analysis on the outage probabilities in Section V. Throughput analysis and simulation results are presented in Section VI-B to examine the effectiveness of the increased levels of coordinations and numbers of active relays for performance enhancement in the three protocols. Concluding remarks are drawn in Section VII.

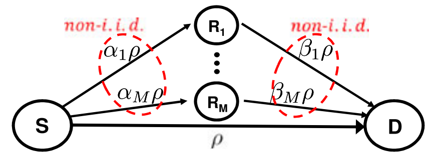

Fig. 1. Source, relays, and the destination, where ρ, $\alpha_m\,\rho$, and $\beta_m\,\rho$ represent the average received SNRs of the S-D, S-R, and R-D radio links, respectively.

II. System Model

We consider a relay-assisted communications system as illustrated in Fig. 1, where there are M relays to help retransmit signals. In the beginning of a packet transmission, the source broadcasts its signal to the relays and the destination. The set of relays that successfully decode the signal is referred to as the decoding set and is denoted by $\mathcal{S}_D$. In case of reception failures at the destination, relays in $\mathcal{S}_D$ are able to jointly retransmit the data with DSTC schemes if $\mathcal{S}_D$ is not an empty set, denoted by $\mathcal{S}_D \neq \emptyset$. Otherwise, the source will rebroadcast the signal until either $\mathcal{S}_D \neq \emptyset$ or the destination is able to successfully decode the signal. To simplify our theoretical investigations, each of the source, destination, and relays are assumed to have one antenna, and the performance metrics are studied from the outage probability point of view. Besides, channels are considered flat faded and complex Gaussian distributed with zero mean and unit variance, denoted by $\sim \mathcal{C}N(0,1)$, and channel coefficients are assumed available at receivers only, and quasi-static, which remain unchanged within the duration of a transmission, and change randomly from one transmission packet to another. In addition, symbol-level synchronization among the source and relays are assumed established. (The same synchronization requirement among the base stations for coordinated multipoint is achieved in the field trial results in [27].) Though simplified, the above system assumptions are valid for fourth-generation (4G) systems [13], [15] that use orthogonal frequency-division multiple access.

According to the above models, let $h_{s,d}$ be the source to the destination (S-D) channel coefficient and h_{s,r_m} be the S-R channel coefficient to relay r_m. Define P_{sd} and P_{sr_m} as the received powers on the respective channels. The corresponding received signals $y_{s,d}$ and y_{s,r_m} can be modeled as

$$y_{s,d} = \sqrt{P_{sd}}h_{s,d}\,x + n_d \tag{1}$$

$$y_{s,r_m} = \sqrt{P_{sr_m}}h_{s,r_m}\,x + n_m, \quad m = 1,2,\ldots,M \tag{2}$$

where the noises n_d at the destination and n_m at relay r_m are $\sim \mathcal{C}N(0, N_0)$. Based on this signal model, the mutual information between the source and relay r_m is given by

$$I_{s,r_m} = \log\left\{1 + P_{sr_m}|h_{s,r_m}|^2/N_0\right\}, \quad m = 1,\ldots,M. \tag{3}$$

Thus, given h_{s,r_m}, the decoding set is more precisely defined as $\mathcal{S}_D \triangleq \{r_m | I_{s,r_m} > R_s, m = 1,\ldots,M\}$, where R_s is the source data rate in bits/sec/channel use, denoted by b/s/cu.

Similarly, one can obtain the mutual information $I_{s,d}$ for the S-D channel and $I_{r,d}$ for the R-D channels. Denote $P_{r_m d}$ as the received power of relayed signals at the destination and assume signals retransmitted by relays in $\mathcal{S}_D$ are coded with orthogonal DSTC, then $I_{r,d}$ can be virtually modeled as [2]

$$I_{r,d} = \log\left\{1 + \sum_{r_m\in\mathcal{S}_D} P_{r_m d}|h_{r_m,d}|^2/N_0\right\}. \tag{4}$$

Based on the system model described above, we present in the next section some capacity outage probabilities to be used in our analysis. For convenience of expression, we denote $[1, M]$ as a set of integers from 1 to M. The received SNRs for the wireless links of the S-D, S-R, and R-D channels are defined as $\rho \triangleq P_{sd}/N_0$, $P_{sr_m}/N_o \triangleq \alpha_m\rho$, and $P_{r_m d}/N_0 \triangleq \beta_m\rho$, respectively, where parameters α_m and β_m characterize the relative radio propagation losses along the S-R and R-D links to that of the S-D link and can be readily obtained from the received signal strength measurements of the corresponding receivers. Following the above assumptions, the probability density function (PDF) of $\rho|h_{s,d}|^2$ is $\lambda\exp\{-\lambda|h_{s,d}|^2\}$ and is denoted by $\sim$Exp(λ), with $\lambda \triangleq 1/\rho$. Similarly, we also have $\alpha_m\rho|h_{s,r_m}|^2 \sim \text{Exp}(\lambda_{1,m})$ and $\beta_m\rho|h_{r_m,d}|^2 \sim \text{Exp}(\lambda_{2,m})\,\forall m \in [1, M]$, with $\lambda_{1,m} \triangleq 1/(\alpha_m\rho)$ and $\lambda_{2,m} \triangleq 1/(\beta_m\rho)$.

III. Capacity Outage Probabilities of Opportunistic Relaying With Orthogonal Distributed Space-Time Coding

Define $W \triangleq \rho|h_{s,d}|^2 \sim \text{Exp}(\lambda)$. Given the source date rate R_s, the outage probability of $I_{s,d} \triangleq \log(1+\rho|h_{s,d}|^2) < R_s$ for the direct S-D channel link, thus, follows:

$$P_W(\delta_s) \triangleq \Pr\{W < \delta_s\} = 1 - e^{-\lambda\delta_s} \tag{5}$$

where $\delta_s \triangleq 2^{R_s} - 1$. Besides, as $\alpha_m\rho|h_{s,r_m}|^2 \sim \text{Exp}(\lambda_{1,m})$, the outage probability of $I_{s,r_m} < R_s$ is similarly given by $1 - e^{-\lambda\delta_s/\alpha_m}$, leading to the probability of $\mathcal{S}_D$ given by

$$\Pr\{\mathcal{S}_D\} = \prod_{m=1,m\in\mathcal{S}_D}^{M} e^{\frac{-\lambda\delta_s}{\alpha_m}} \prod_{m'=1,m'\notin\mathcal{S}_D}^{M}\left(1 - e^{\frac{-\lambda\delta_s}{\alpha_{m'}}}\right). \tag{6}$$

Further, define $X_m \triangleq \beta_m\rho|h_{r_m,d}|^2 \sim \text{Exp}(\lambda_{2,m})$, and $\mathcal{X} \triangleq \{X_m | m \in \mathcal{S}_D\}$, and assume that $\mathcal{X}$ is made available to the destination. According to the rule of OMPR [28], we define that a maximum of i relays with the i largest X_m's in $\mathcal{X}$ can be chosen by the destination to help forward the source signal. The number of chosen relays is, thus, written as $i_{\mathcal{D}} \triangleq \min\{i, \mathcal{D}\}$, where $\mathcal{D} \triangleq |\mathcal{S}_D|$ stands for the cardinality of $\mathcal{S}_D$. Given that the relays are synchronized in advance, the channels between the transmitting relays and the destination can be viewed as a MISO channel. Sorting the elements of $\mathcal{X}$ in the ascending order into $\mathcal{X}' \triangleq \{X'_1,\ldots,X'_{\mathcal{D}}\}$ with $X'_k \geq X'_j$ for $k > j$, then, conditioned on $\mathcal{D} = d$, the outage probability of $I_{r,d} < R_r$ of using orthogonal DSTC for such an OMPR scheme (ODSTCi) is, thus, given by

$$P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d) \triangleq \Pr\left\{\sum_{j=\max\{1,d-i+1\}}^{d} X'_j < \delta_r \,\middle|\, d \geq 1\right\} \tag{7}$$

where $\delta_r \triangleq 2^{R_r} - 1$ and R_r is the forwarding data rate. The derivation of (7) is provided in Appendix A.

Based on the above forwarding rule, the outage probability after one time retransmission is given by

$$\begin{aligned}\mathbb{P}_{\mathcal{O}_i} &= P_W^2(\delta_s)\Pr\{\mathcal{S}_D = \emptyset\}\\ &\quad + P_W(\delta_s)\sum_{d=1}^{M} P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)\Pr\{|\mathcal{S}_D| = d\}\end{aligned} \tag{8}$$

where the first term on the right-hand side (RHS) corresponds to the case of source retransmission when $\mathcal{S}_D = \emptyset$. The results of (8) for $R_s = R_r = 4$, $M = 4$, and $i = 2$ are shown in Fig. 2(a). The received SNRs of the S-R and R-D channels are set as $\alpha_m\rho$ and $\beta_m\rho$, respectively, according to the SNR ρ of the direct link, with $\bar{\alpha} \triangleq [\alpha_1, \alpha_2, \alpha_3, \alpha_4] = [110, 105, 95, 90]$ and $\bar{\beta} \triangleq [\beta_1, \beta_2, \beta_3, \beta_4] = [5, 8, 12, 15]$. We note that relays in the

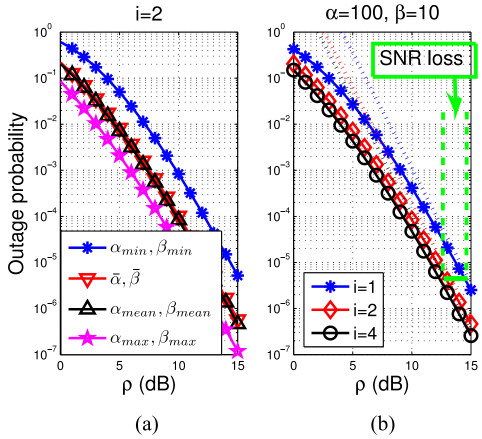

Fig. 2. Outage probabilities with ODSTCi relaying when $R_s = R_r = 4$ and $M = 4$.

same relaying group are considered close to each other such that α_m and β_m are also close to each other in $\bar{\alpha}$ and $\bar{\beta}$.

Also shown in Fig. 2(a) are the results of which all α_m and β_m are set to their corresponding maximum and minimum values. The results, thus, form an upper and a lower bounds of the true outage probabilities. It is interesting to see that the true outage probabilities are also close to the ones in which all α_m and β_m are set to their respective means. This allows us to proceed the following analysis with an approximate but much simpler system setting of $\alpha_m = \alpha$ and $\beta_m = \beta\ \forall m$. As such, we have $\alpha_m \rho |h_{s,r_m}|^2 \sim \text{Exp}(\lambda_1)$ with $\lambda_1 = 1/\alpha\rho$, and $\beta_m \rho |h_{r_m,d}|^2 = X_m \sim \text{Exp}(\lambda_2)$ with $\lambda_2 = 1/\beta\rho\ \forall m$. Accordingly, (7) can be evaluated immediately by replacing $\lambda_{2,m}$ with λ_2 in (60) in Appendix A, leading to

$$P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d) = 1 - \sum_{k=0}^{i_d-1} e^{-\delta_r\lambda_2}\frac{(\delta_r\lambda_2)^k}{k!} - \sum_{j=1}^{d-i_d} e^{-\frac{\tilde{c}_j}{i_d}\delta_r\lambda_2} \times \frac{\tilde{a}_j}{(-\tilde{b}_j)^{i_d}}\left[1 - e^{\tilde{b}_j\delta_r\lambda_2}\sum_{\ell=0}^{i_d-1}\frac{(-\tilde{b}_j\delta_r\lambda_2)^\ell}{\ell!}\right] \tag{9}$$

with $\tilde{a}_j(i) \triangleq \frac{1}{d-j+1}\frac{d!}{i!}\frac{(-1)^{d-i-j}}{(j-1)!(d-i-j)!}$, $\tilde{b}_j(i) \triangleq \frac{d-i-j+1}{i}$, and $\tilde{c}_j(i) \triangleq d - j + 1$.

Similarly, the outage probability of (8) in this case degenerates to

$$\mathbb{P}_{\mathcal{O}_i} = P_W^2(\delta_s)\mathcal{P}_\mathcal{D}(0) + P_W(\delta_s)\sum_{d=1}^{M} P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)\mathcal{P}_\mathcal{D}(d) \tag{10}$$

where the probability mass function (PMF) of the cardinality $\mathcal{D}$ of $\mathcal{S}_D$ is given by

$$\mathcal{P}_\mathcal{D}(d) = C_d^M(e^{-\delta_s/\alpha})^d(1 - e^{-\delta_s/\alpha})^{M-d} \tag{11}$$

with C_d^M standing for the number of combinations of randomly choosing d out of M.

For convenience of expression, and to distinguish from the ordinary DSTC protocol, we use ODSTCi in the sequel to signify the use of maximally i active relays for opportunistic DSTC in retransmissions. For the case of $i = 1$, the outage probability is equal to that of the OR in [10], while for ODSTCM, it is equal to the typical DSTC relaying in [2]. The results of (10) are provided in Fig. 2(b), and the dashed lines represent the high SNR approximations. It is clear to see that the outage probabilities of the cases of $i = 1$, $i = 2$, and $i = M$ have the same slope with some SNR offsets that are marked with green lines in Fig. 2(b). These SNR offset gains come from the individual power of the active relays, whose analysis will be given in the next section.

A. Relay Efficiency of Cooperative Relaying via ODSTC

To motivate the subsequent performance analysis throughout the paper, we would first characterize the asymptotic outage probability of the ODSTCi scheme that uses maximally i relays in $\mathcal{S}_D$ for cooperative relaying. The analysis is carried out at high SNR for us to quantify the *relay efficiency* on the SNR gain per active relay for ODSTCi versus the DSTC (ODSTCM) that uses all relays in $\mathcal{S}_D$ and the OR (ODSTC1) that uses only one relay in $\mathcal{S}_D$ for signal forwarding.

To begin with, we first provide the asymptotic functions of $P_W(\delta_s)$, $\mathcal{P}_\mathcal{D}(d)$, and $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)$ at high SNR. For $P_W(\delta_s)$ of (5) and $\mathcal{P}_\mathcal{D}(d)$ of (11), it has been shown in [2] that

$$\lim_{\lambda\to 0}\frac{P_W(\delta_s)}{\lambda} = \delta_s \text{ and } \lim_{\lambda\to 0}\frac{\mathcal{P}_\mathcal{D}(d)}{\lambda^{M-d}} = \frac{C_d^M\delta_s^{M-d}}{\alpha^{M-d}} \tag{12}$$

where $\rho \triangleq 1/\lambda$. As for $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)$, we derive in Appendix B a property stated below.

Proposition 1: Given δ_r, β, and $i_d \triangleq \min\{i, d\}$, it follows that

$$\lim_{\lambda\to 0}\left\{\frac{P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)}{\lambda^d}\right\} = \frac{\delta_r^d}{i_d!i_d^{d-i_d}\beta^d}. \tag{13}$$

Now, we define an asymptotic function of $f(\lambda)$ as $a\lambda^n$, denoted by $f(\lambda) \doteq a\lambda^n$, if $a \neq 0$, $\lambda \ll 1$, and $\lim_{\lambda\to 0}\{f(\lambda)/\lambda^n\} = a$ for $n \geq 0$ [29]. Following this definition, it is straightforward to show that $a\lambda^n \pm b\lambda^m \doteq a\lambda^n$, if $m > n \geq 0$, and $a\lambda^n \cdot b\lambda^m \doteq ab\lambda^{n+m}$, if $m + n \geq 0$. Accordingly, we have $P_W(\delta_s) \doteq \delta_s/\rho$ and $\mathcal{P}_\mathcal{D}(d) \doteq C_d^M(\delta_s/\alpha\rho)^{M-d}$ when the SNR $\rho = 1/\lambda \gg 1$. Furthermore, by Proposition 1, we also have

$$P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d) \doteq \frac{\delta_r^d\lambda^d}{i_d!i_d^{d-i_d}\beta^d} \equiv \frac{\delta_r^d}{i_d!i_d^{d-i_d}\beta^d\rho^d}. \tag{14}$$

Applying the results of (12)∼(14) back into (10) yields

$$\mathbb{P}_{\mathcal{O}_i} \doteq \frac{\delta_s^{M+1}}{\alpha^M\rho^{M+1}}\left[\frac{\delta_s}{\rho} + \sum_{d=1}^{M}\frac{C_d^M}{i_d!i_d^{d-i_d}}\left(\frac{\alpha\delta_r}{\beta\delta_s}\right)^d\right]. \tag{15}$$

The two terms on the RHS are, respectively, the asymptotic functions of the two terms on the RHS of (10). If ρ is large enough such that $\frac{\delta_s}{\rho} \ll \sum_{d=1}^{M}\frac{C_d^M}{i_d!i_d^{d-i_d}}(\frac{\alpha\delta_r}{\beta\delta_s})^d$, we have

$$\mathbb{P}_{\mathcal{O}_i} \doteq \frac{\delta_s^{M+1}}{\alpha^M\rho^{M+1}}\sum_{d=1}^{M}\frac{C_d^M}{i_d!i_d^{d-i_d}}\left(\frac{\alpha\delta_r}{\beta\delta_s}\right)^d \triangleq \widetilde{\mathbb{P}}_{\mathcal{O}_i}. \tag{16}$$

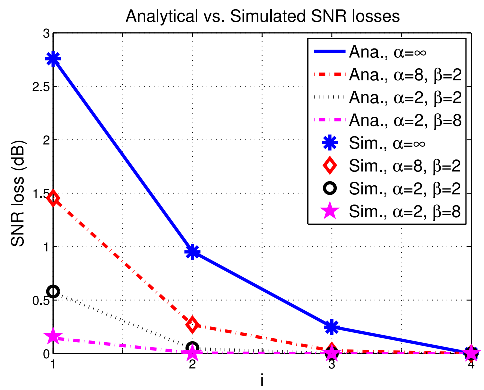

Fig. 3. SNR losses $\mathcal{L}(i)$ of ODSTCi for $R_s = R_r = 2$ and $M = 4$.

Following the diversity definition provided in [29], we thus have the diversity of $\mathbb{P}_{\mathcal{O}_i}$ as

$$\xi \triangleq -\lim_{\rho\to\infty}\left\{\frac{\log \mathbb{P}_{\mathcal{O}_i}}{\log \rho}\right\} = M+1. \tag{17}$$

Equation (17) implies that every ODSTCi scheme, $i = 1,\ldots, M$, achieves the same diversity as expected in Fig. 2(b), while offers different coding gains. The asymptotic coding gain of ODSTCi relaying, denoted by G_i, is defined at high SNR as

$$G_i \triangleq \lim_{\rho\to\infty}\frac{\mathbb{P}_{\mathcal{O}_i}}{\rho^{-\xi}} = \frac{\widetilde{\mathbb{P}}_{\mathcal{O}_i}}{\rho^{-\xi}} = \frac{\delta_s^{M+1}}{\alpha^M}\sum_{d=1}^{M}\frac{C_d^M}{i_d!\,i_d^{d-i_d}}\left(\frac{\alpha\delta_r}{\beta\delta_s}\right)^d. \tag{18}$$

Based on (18), the SNR loss of using ODSTCi than using DSTC can be quantified at high SNR as the extra SNR required by the ODSTCi to achieve the same outage probability that is achieved with the DSTC. Namely, given a target outage probability $\mathbb{P}'$ such that $\mathbb{P}' = G_i\rho_i^{-\xi} = G_M\rho_M^{-\xi}$, the SNR loss of ODSTC$i$ is defined in the logarithmic scale as

$$\mathcal{L}(i) \triangleq \log\{\rho_i\} - \log\{\rho_M\} = \frac{\log\{G_i\} - \log\{G_M\}}{\xi}. \tag{19}$$

B. Numerical Studies

The outage probabilities of (10) and their asymptotic functions of (16) for some values of i are shown in Fig. 2(b). Clearly, the dotted lines of the asymptotic functions match their corresponding exact curves of (10) at high SNR. Besides, the advantage of using more relays diminishes rapidly when i increases. The analytical SNR losses $\mathcal{L}(i)$ of (19) versus their simulated counterparts are presented in Fig. 3 for various sets of $\{\alpha, \beta\}$ when $M = 4$ and $R_s = R_r = 2$. As shown in the figure, when the S-R to S-D link ratio α is higher than the R-D to S-D link ratio β, the SNR losses $\mathcal{L}(i)$ are more pronounced when the number of active relays i decreases. In fact, if α continues to increase such that $\alpha \gg \beta$, then $\mathbb{P}_{\mathcal{O}_i}$ of (15) is eventually dominated by the event of $\mathcal{D} = M$, which makes $\mathbb{P}_{\mathcal{O}_i} \doteq \frac{1}{i!i^{M-i}}\frac{\delta_s}{\rho}\frac{\delta_r^M}{\beta^M\rho^M}$. Then, it follows directly from (17) to (19) that

$$\mathcal{L}(i) \cong \frac{1}{M+1}\log\left\{\frac{M!}{i!i^{(M-i)}}\right\}, \quad \text{if } \alpha \gg \beta. \tag{20}$$

This shows that when the relay channel resembles a MISO one as $\alpha \gg \beta$, $\mathcal{L}(i)$ becomes irrelevant of α and β, and decreases rapidly with respect to (w.r.t.) i as verified in Fig. 3 when $\alpha \to \infty$. In contrast, if $\beta > \alpha$, then $\mathcal{L}(i)$ becomes very small $\forall i \in [1, M]$. In fact, if $\beta \gg \alpha$ such that $\mathbb{P}_{\mathcal{O}_i}$ is dominated by the event of $\mathcal{D} = 1$ in (15), then $\widetilde{\mathbb{P}}_{\mathcal{O}_i} \cong \frac{M}{\rho^{M+1}}\frac{\delta_s^M}{\alpha^{M-1}}\frac{\delta_r}{\beta}$ and, hence, $\mathcal{L}(i) \cong 0$ by (18) and (19). Under this channel condition, the destination can decode the signal as long as any one of the relays can do so. The OR ($i = 1$) scheme appears to be the best choice in this type of channels.

IV. Capacity Outage Probabilities of Relay-Assisted ARQ Protocols With Opportunistic Multipoint Relaying and Distributed Space-Time Coding

Based on the analysis for ODSTCi relaying, we introduce three representative methods to apply ODSTCi for relay-assisted ARQ, and present their outage probability analysis. Each method requires a different level of coordinations among the source, relays, and destination to exploit the potential spatial and temporal diversities in ARQ with ODSTCi relaying.

A. Type-A ARQ Protocol With One-Time Relay Selection

As introduced earlier, when outages occur and $\mathcal{S}_D = \emptyset$, the destination will request a packet retransmission from the source. Once $\mathcal{S}_D \neq \emptyset$, by ODSTC$i$, the best $i_{\mathcal{D}} = \min\{i, \mathcal{D}\}$ relays $\in \mathcal{S}_D$ that maximize (4) will be chosen for retransmission. Then, the simplest way for the subsequent ARQs of the same packet is to continue using the same relays for retransmissions. This type of ARQ method is referred to as the Type-A protocol herein, and essentially involves two relaying methods: the ODSTCi when $\mathcal{S}_D$ just turns nonempty, and an ordinary DSTC that uses the same $i_{\mathcal{D}}$ relays in the subsequent ARQs. The reason is that the R-D channel coefficients change independently in every packet transmission, making the opportunistic diversity from channel ordering no longer available in the subsequent ARQs of the same packet. In the sequel, for clarity, we denote the DSTC by DSTCi to indicate the maximum number of active relays for retransmissions.

The outage probability of ODSTCi has been provided in (9), and the outage probability of DSTCi can be obtained by setting d to i_d in the RHS of (9), yielding

$$P_{D_i|\mathcal{D}}(\delta_r|d) = 1 - \sum_{j=0}^{i_d-1}\frac{e^{-\delta_r\lambda_2}}{j!(\delta_r\lambda_2)^{-j}}. \tag{21}$$

Based on $P_W(\delta_s)$, $\mathcal{P}_{\mathcal{D}}(d)$, $P_{O_i|\mathcal{D}}(\delta_r|d)$, and $P_{D_i|\mathcal{D}}(\delta_r|d)$ of (5), (11), (7), and (21), respectively, we are ready to analyze the outage probability of the Type-A protocol, which is provided in the following proposition.

Proposition 2: Given R_s, R_r, M, and $i \in [1, M]$, the outage probability after n rounds of retransmissions of the Type-A

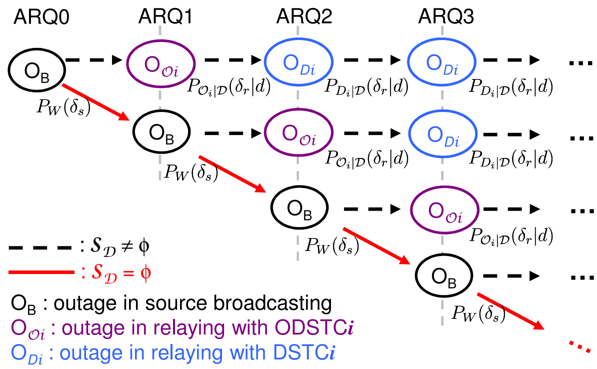

Fig. 4. Outage events in the Type-A ARQ protocol.

protocol is given by

$$\mathbb{P}_{A,i}(n) = P_W^{n+1}(\delta_s)\mathcal{P}_\mathcal{D}^n(0) + \sum_{k=1}^{n}\sum_{d=1}^{M} P_W^{n-k+1}(\delta_s)\mathcal{P}_\mathcal{D}^{n-k}(0) \times \mathcal{P}_\mathcal{D}(d)P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)P_{D_i|\mathcal{D}}^{k-1}(\delta_r|d) \tag{22}$$

with $\mathbb{P}_{A,i}(0) \triangleq P_W(\delta_s)$ in (5).

Proof: We use a tree diagram in Fig. 4 to illustrate the outage events that might occur in the Type-A protocol. For simplicity, we denote the nth round of retransmissions by ARQn and refer the initial transmission from the source as ARQ0. The ellipses marked by O_B stand for the outage events in source broadcasting (BC), whose probability is given by $P_W(\delta_s)$. Starting from the upper left of Fig. 4, it shows that if an outage occurs after the source BC and $\mathcal{S}_D \neq \emptyset$, i.e., $\mathcal{D} \neq 0$, then the retransmission that follows immediately after the BC will employ the ODSTCi, whose outage event is marked by $\mathrm{O}_{\mathcal{O}i}$ in Fig. 4 with the probability given by $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)$. In case the retransmission still fails, the same relays will keep retransmitting with DSTCi whose outage events are marked as O_{Di}, with a probability equal to $P_{D_i|\mathcal{D}}(\delta_r|d)$ of (21). The time sequence of these outage events is shown as a row of ellipses linked by dashed arrows from left to right.

On the other hand, if $\mathcal{S}_D = \emptyset$ after BC, then the source will rebroadcast the signal, which might be followed by another source BC if an outage event still happens and $\mathcal{S}_D = \emptyset$, or by another sequence of retransmissions the same to what are described above for $\mathcal{S}_D \neq \emptyset$ if outages continue to happen. Since $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)$ and $P_{D_i|\mathcal{D}}(\delta_r|d)$ condition on $\mathcal{D}$ only and not on any specific relay in $\mathcal{S}_D$, it is clear from the tree diagram of Fig. 4 that the outage probability has a recursive form of

$$\mathbb{P}_{A,i}(n) = P_W(\delta_s)\sum_{d=1}^{M}\mathcal{P}_\mathcal{D}(d)P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)P_{D_i|\mathcal{D}}^{n-1}(\delta_r|d) + P_W(\delta_s)\mathcal{P}_\mathcal{D}(0)\mathbb{P}_{A,i}(n-1). \tag{23}$$

Expanding this recursive form directly gives (22). ■

B. Type-B ARQ Protocol With Relay Reselection

Though simple, the diversity of the Type-A protocol might be very limited after ARQ2 as the opportunistic diversity is no longer available due to the fixed relaying thereafter. A quick modification to resolve this problem is to have the i_d active relays be rechosen from $\mathcal{S}_D$ according to the channel strength in each round of ARQ. The relay reselection procedure for the subsequent relay retransmissions is the same to that for the first one. We refer to this type of ARQ as the Type-B protocol. Due to this reselection mechanism, we know that $P_{D_i|\mathcal{D}}(\delta_r|d)$ in (22) for the Type-A protocol should be replaced by $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)$ in this case. This results in the outage probability below for the Type-B protocol.

Corollary 1: Given R_s, R_r, M, and $i \in [1, M]$, the outage probability after n rounds of retransmissions of the Type-B protocol is given by

$$\mathbb{P}_{B,i}(n) = P_W^{n+1}(\delta_s)\mathcal{P}_\mathcal{D}^n(0) + \sum_{k=1}^{n}\sum_{d=1}^{M} P_W^{n-k+1}(\delta_s)\mathcal{P}_\mathcal{D}^{n-k}(0) \times P_{\mathcal{O}_i|\mathcal{D}}^k(\delta_r|d)\mathcal{P}_\mathcal{D}(d) \tag{24}$$

with $\mathbb{P}_{B,i}(0) \triangleq P_W(\delta_s)$.

In contrast to the Type-A protocol, the Type-B requires all relays in $\mathcal{S}_D$ to keep the decoded packet data for future reselections and retransmissions until the destination succeeds in decoding. However, checking (24), one may soon find that the diversity order might still be limited by the event of $\mathcal{D} = 1$ if $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|1)$ dominates the other $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)\forall d > 1$. This situation may happen when the link quality between the source and relays is poor, and would make the Type-B protocol rather ineffective, taking into account the extra efforts to reselect the relays in each retransmission. A quick remedy to this diversity shortage problem is to allow overhearing on the relays $\notin \mathcal{S}_D$.

C. Type-C ARQ Protocol With Relay Overhearing

We refer to this type of ARQ as the Type-C protocol, and have relays $\notin \mathcal{S}_D$ continue to overhear the signals sent by the chosen relays and update their statuses to the destination to allow being selected in the subsequent retransmissions. To begin with, we first characterize the outage probability for relays that overhear the DSTC signals sent by the chosen relays. The signals x_m sent from relay r_m and received at relay r_ℓ, for $r_m \in \mathcal{S}_D$ and $r_\ell \notin \mathcal{S}_D$, are modeled as [7]

$$y_{r_m,r_\ell} = \sqrt{P_{rr}}h_{r_m,r_\ell}x_m + n_\ell \tag{25}$$

where h_{r_m,r_ℓ} represent the channel coefficients between the relays r_m and r_ℓ. For convenience of exposition, channels between any transmit-and-receive pairs of the relays (r_m, r_ℓ) are assumed to have the same received SNR, $P_{rr}/N_0 \triangleq \eta\rho$, but individual channel coefficients $\eta\rho|h_{r_m,r_\ell}|^2 \sim \mathrm{Exp}(\lambda_3)$ with $\lambda_3 \triangleq 1/(\eta\rho)$. Since the channels between different pairs of (r_m, r_ℓ) fade independently, the channels from the relays, r_m, which are particularly chosen from $\mathcal{S}_D$ to yield the highest mutual information at the destination, do not provide the overhearing relays r_ℓ, the same opportunistic diversity from channel ordering. Therefore, the outage probability of overhearing has the ordinary form of MISO channel given in (21). Define $V_{m,\ell} \triangleq \eta\rho|h_{r_m,r_\ell}|^2 \sim \mathrm{Exp}(\lambda_3)$ and $\mathcal{V}_i \triangleq \sum_{m=1}^{i_d} V_{m,\ell}$. The outage probability for a relay ℓ to overhear the DSTC signal send by i_d active relays in $\mathcal{S}_D$ is, thus, given by

$$P_{\mathcal{V}_i|\mathcal{D}}(\delta_r|d) \triangleq \Pr\left\{\mathcal{V}_i < \delta_r \,\middle|\, d \geq 1\right\} = 1 - \sum_{j=0}^{i_d-1} \frac{e^{-\delta_r\lambda_3}}{j!(\delta_r\lambda_3)^{-j}}. \tag{26}$$

Based on this result, we shall reformulate the PMF of $\mathcal{S}_D$ for $|\mathcal{S}_D|$ to continue increasing with retransmissions in the Type-C protocol. First, we define some RVs below.

Definition 1: Let $\mathcal{D}_0 \in [0, M]$ be the number of relays that decode the signal from the source.

Definition 2: Let $\mathcal{D}_n \in [0, M]$ be the number of increasing relays in the nth retransmission after $\mathcal{D}_0 \geq 1$.

Definition 3: Let $\underline{\mathcal{D}}_n = \sum_{\ell=0}^{n} \mathcal{D}_\ell$ and $\underline{\mathcal{D}}_n \in [1, M]$ be the total number of relays that are able to decode the signal up to the nth retransmission after $\mathcal{D}_0 \geq 1$.

According to these definitions, it follows directly from (11) that $\mathcal{P}_{\mathcal{D}_0}(d_0) = C_{d_0}^{M} e^{-\lambda_1\delta_s d_0}(1 - e^{-\lambda_1\delta_s})^{M-d_0}$. Besides, as the source stops sending signal once $\mathcal{D}_0 \geq 1$. By (26), we have the PMF of $\mathcal{D}_n$ conditioned on $\underline{\mathcal{D}}_{n-1}$ given by

$$\begin{aligned}\mathcal{P}_{\mathcal{D}_n|\underline{\mathcal{D}}_{n-1}}(d_n|\underline{d}_{n-1}) = C_{d_n}^{M-\underline{d}_{n-1}}&[1 - P_{\mathcal{V}_i|D}(\delta_r|\underline{d}_{n-1})]^{d_n}\\ &\times [P_{\mathcal{V}_i|D}(\delta_r|\underline{d}_{n-1})]^{M-\underline{d}_n}, \ n = 1, 2, \ldots\end{aligned} \tag{27}$$

where $\underline{d}_n \triangleq \sum_{\ell=0}^{n} d_\ell$. Given the conditional PMF of $\mathcal{D}_n$, the capacity outage probability of the Type-C protocol can thus be provided in the following proposition.

Proposition 3: Given R_s, R_r, M, and $i \in [1, M]$, the capacity outage probability after n rounds of retransmissions of the Type-C protocol is given by (28), shown below, where $\underline{d}_k \triangleq \sum_{q=0}^{k} d_q$, $\mathbb{P}_{C,i}(0) \triangleq P_W(\delta_s)$, and $\mathbb{P}_{C,i}(1) \triangleq \mathbb{P}_{\mathcal{O}_i}$:

$$\begin{aligned}\mathbb{P}_{C,i}(n) = P_W^{n+1}(\delta_s)\mathcal{P}_{\mathcal{D}_0}^{n}(0) &+ \sum_{k=1}^{n} P_W^{n-k+1}(\delta_s)\mathcal{P}_{\mathcal{D}_0}^{n-k}(0)\\ &\times \sum_{d_0=1}^{M}\sum_{d_1=0}^{M-d_0}\cdots\sum_{d_{k-1}=0}^{M-\underline{d}_{k-2}} P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d_0)\mathcal{P}_{\mathcal{D}_0}(d_0)\\ &\times \prod_{\ell=1}^{k-1} P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|\underline{d}_\ell)\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1}).\end{aligned} \tag{28}$$

Proof: The proof follows a similar approach used in proving Proposition 2. Again, the outage events that may occur in the Type-C protocol are illustrated as a tree diagram in Fig. 5 . If $\mathcal{D}_0$ turns nonzero at the $(n-k)$th retransmission, and an outage still happens, then ODSTCi will start to be used for retransmission by opportunistically choosing $\min\{i, \underline{\mathcal{D}}_0\}$ relays out of $\mathcal{S}_D$ that maximize (4) at the end of the $(n-k)$th retransmission. It is noted that $\mathcal{S}_D$ will continue to grow due to the overhearing mechanism.

If outage events still continue to happen, then ODSTCi will continue to be used for retransmissions as well, by re-choosing

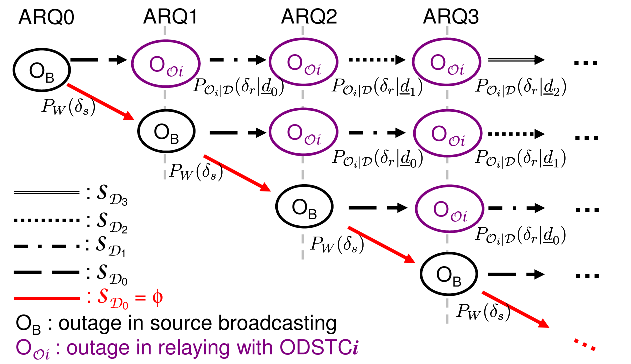

Fig. 5. Outage events in the Type-C ARQ protocol.

$\min\{i, \underline{\mathcal{D}}_p\}$ relays from $\mathcal{S}_D$ at the end of the $(n-k+p)$th retransmission. The time sequence of these outage events is shown as a row of ellipses linked by dashed or dotted arrows from left to right in Fig. 5. The arrows of different types represent different numbers of retransmissions after $\mathcal{D}_0 \geq 1$, thus corresponding to different cardinalities, $\underline{\mathcal{D}}_p$, of $\mathcal{S}_D$. Since $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ conditions on $|\mathcal{S}_D|$ only, from Fig. 5, the capacity outage probability of the Type-C protocol has a recursive form

$$\begin{aligned}\mathbb{P}_{C,i}(n) &= P_W(\delta_s)\mathcal{P}_{\mathcal{D}_0}(0)\mathbb{P}_{C,i}(n-1)\\ &\quad + P_W(\delta_s)\sum_{d_0=1}^{M}\cdots\sum_{d_{n-1}=0}^{M-\underline{d}_{n-2}} P_{\mathcal{O}_i|\mathcal{D}}(\delta|d_0)\mathcal{P}_{\mathcal{D}_0}(d_0)\\ &\quad \times \prod_{\ell=1}^{n-1} P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_\ell)\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1}).\end{aligned} \tag{29}$$

Expanding this recursive form directly gives (28). ■

V. Asymptotic Outage Probability and Relay Efficiencies in the Three ARQ Protocols

To study the diversities of the different ARQ protocols and their relay efficiencies in the SNR gains per active relays i for ODSTCi, we first characterize the asymptotic properties of their capacity outage probabilities under different code rates in δ_s and δ_r, and channel conditions parameterized by α, β, and η. The results are then used in the analysis of relay efficiencies.

A. Asymptotic Outage Probability of the Type-A Protocol

To analyze the asymptotic function of (22) of the Type-A protocol, it is also necessary to obtain the asymptotic function of $P_{D_i|\mathcal{D}}(\delta|d)$ at high SNR. By Proposition 1, the asymptotic function of $P_{D_i|\mathcal{D}}(\delta|d)$ of (21) can be readily obtained by setting $d = i_d$ in (14), which gives

$$P_{D_i|\mathcal{D}}(\delta_r|d) \doteq \frac{\delta_r^{i_d}}{i_d!\beta^{i_d}\rho^{i_d}} \triangleq \frac{p_{rd}^{i_d}}{i_d!} \tag{30}$$

where $p_{rd} \triangleq \frac{\delta_r}{\beta\rho} < 1$. Together with (12) and (14), the asymptotic function of (22) can be written as

$$\mathbb{P}_{A,i}(n) \doteq p_{sd}\Lambda_{\alpha,\rho}^n + \sum_{k=1}^{n}\sum_{d=1}^{M}\Omega_{d,k}\Lambda_{\alpha,\rho}^{n-k+1}\left(\frac{p_{rd}}{p_{sr}}\right)^d p_{rd}^{i_d(k-1)} \tag{31}$$

where $\Omega_{d,k} \triangleq \frac{C_d^M}{i_d^{(d-i_d)}(i_d!)^k}$ and $\Lambda_{\alpha,\rho} \triangleq p_{sd}p_{sr}^M$ with $p_{sd} \triangleq \frac{\delta_s}{\rho} < 1$ and $p_{sr} \triangleq \frac{\delta_s}{\alpha\rho} < 1$.

We may regard p_{sd}, p_{sr}, and p_{rd} as the respective probabilities of the S-D, S-R, and R-D links failures. It is, however, not straightforward to analyze $\mathbb{P}_{A,i}(n)$ under the different operating conditions and geometric relationships among the source, relays, and the destination. We summarize our analysis in the next proposition with details relegated in Appendix D.

Proposition 4: Suppose $p_{sr} < p_{sd} < 1$ and $p_{rd} < p_{sd} < 1$, namely both α and $\beta > 1$, the asymptotic function of $\mathbb{P}_{A,i}(n)$ of the Type-A protocol with ODSTCi relaying is given by

1) For $2 \le i \le M$, we have

$$\mathbb{P}_{A,i}(n) \cong \begin{cases} p_{sd}^{n+1}p_{sr}^{nM}, & p_{sr} > \bar{p}_{rd} \\ p_{sd}p_{sr}^{M}p_{rd}^{i(n-1)}\sum_{d=i}^{M}\Omega_{d,n}\left(\frac{p_{rd}}{p_{sr}}\right)^d, & p_{rd} \ge \bar{p}_{sr} \\ Mp_{sd}p_{sr}^{M-1}p_{rd}^n, & \text{otherwise} \end{cases} \tag{32}$$

where $\bar{p}_{rd} \triangleq \max\{p_{rd}, p_{A_2}\}$ and $\bar{p}_{sr} \triangleq \max\{p_{sr}^{\frac{1}{n}}, p_{A_1}\}$ with $p_{A_1} = [\sum_{d=i}^{M}\frac{\Omega_{d,n}}{M}(\frac{\alpha}{\delta_s}\frac{\delta_r}{\beta})^{d-1}]^{\frac{-1}{(n-1)(i-1)}}$ and $p_{A_2} = [\frac{1}{M}(\frac{\delta_s}{\delta_r}\beta)^n]^{\frac{-1}{M(n-1)+1}}$.

2) For $i = 1$, however, we have

$$\mathbb{P}_{A,1}(n) \cong \begin{cases} p_{sd}^{n+1}p_{sr}^{nM}, & p_{sr} > \tilde{p}_{rd} \\ p_{sd}p_{sr}^{M}p_{rd}^{n-1}\sum_{d=1}^{M}\Omega_{d,n}\left(\frac{p_{rd}}{p_{sr}}\right)^d, & \text{otherwise} \end{cases} \tag{33}$$

with $\tilde{p}_{rd} = \max\{p_{rd}, p_{A_3}\}$, $p_{A_3} \triangleq [\frac{1}{\Delta_{\alpha,\beta}}(\frac{\delta_s}{\delta_r}\beta)^n]^{\frac{-1}{M(n-1)+1}}$, and $p_{A_3} \ge p_{A_2}$ since $\Delta_{\alpha,\beta} \triangleq \sum_{d=1}^{M}C_d^M(\frac{\alpha}{\delta_s}\frac{\delta_r}{\beta})^{d-1} \ge M$.

The results show that the diversity of the Type-A protocol is $\xi_A = M + n$ by the definition of (17) when $\rho \to \infty$. For finite SNR, however, the decreasing rate of the asymptotic function w.r.t. ρ may vary under the different settings of δ_s and δ_r, and the link qualities α and β on the S-R and R-D channels. When $p_{sr} \le p_{rd}^n$, which means the probability of the S-R link failure is much less than the probability of the R-D link failure, more active relays, namely, a larger value of i, for ODSTCi results in a higher decreasing rate, which is proportional to $(\frac{1}{\rho})^{M+1+i(n-1)}$, on the asymptotic function when $\rho \le \frac{\delta_r}{\beta}p_{A_1}^{-1}$, i.e., $p_{rd} \ge p_{A_1}$, as well. Nevertheless, one can also see from (31) that $p_{rd}^{i(k-1)}$ for the events of $d \ge i$ and, hence, $i_d = i$, drops far more quickly than $p_{rd}^{d(k-1)}$ for the events of $d < i$ and $i_d = d$ as p_{rd} decreases, i.e., ρ increases. Eventually, when $p_{rd} < p_{A_1}$, i.e., $\rho > \frac{\delta_r}{\beta}p_{A_1}^{-1}$, the second term of (31) becomes dominated by the event of $d = 1 = i_d$, resulting in a diversity order of $M + n$ at high SNR. Namely, the dominant order of $\frac{1}{\rho}$ converges from $M + 1 + i(n-1)$ to $M + n$ when $\rho > \frac{\delta_r}{\beta}p_{A_1}^{-1}$. This *diversity loss* phenomenon can be delayed to occur at a higher SNR with a larger value of $\frac{\alpha}{\delta_s}$ as suggested in the form of p_{A_1} in (32). If we have $\frac{\alpha}{\delta_s} \to \infty$, then the chance for $\mathcal{D} < M$ will become zero, leading to an $M \times 1$ virtual MISO channel with antenna selection. The diversity will remain at $M + 1 + i(n-1)$ in this case when $\rho \to \infty$.

On the other hand, when $p_{sr} > p_{rd}$, the probability of the S-R link failure becomes greater than the probability of the R-D link failure. The relays become less successfully decoding their received packets than the destination decoding their forwarded packets. In case all relays and the destination fail to decode a packet, the source will retransmit the packet by itself. The probability of this event is asymptotically equal to $\Lambda_{\alpha,\rho} \triangleq p_{sd}p_{sr}^M$ in (31). The resultant outage probability due to this kind of events is lumped in the first term on the RHS of (31), which, however, drops far more quickly than the second term of (31) as ρ increases. Eventually, when $\rho > \frac{\delta_s}{\alpha}p_{A_2}^{-1}$, i.e., $p_{sr} < p_{A_2}$, (31) becomes dominated by the event of $\mathcal{D} = 1$ in the second term, and changes from the order of $\frac{1}{\rho^{n(M+1)+n}}$ to the order of $\frac{1}{\rho^{M+n}}$ as it shows in (32). Similarly, this diversity loss phenomenon can be delayed to occur at a higher SNR with a larger value of $\frac{\beta}{\delta_r}$ as suggested in the form of p_{A_2} in (32). If we have $\frac{\beta}{\delta_r} \to \infty$, then the destination can decode the relayed packet when any of the relays can decode and forward it. This corresponds to a $1 \times (M+1)$ virtual SIMO channel with individual decoder on each channel output. The diversity will remain at $n(M+1)+1$ in this case when $\rho \to \infty$.

For the case of $i = 1$, the arguments for $p_{sr} \le p_{rd}^n$ no longer apply since the diversity is always equal to $M + n$. While for $p_{sr} > p_{rd}$, similar arguments apply for $\rho \le \frac{\delta_s}{\alpha}p_{A_3}^{-1} \le \frac{\delta_s}{\alpha}p_{A_2}^{-1}$.

B. Asymptotic Outage Probability of the Type-B Protocol

For the asymptotic function of the outage probability (24) of the Type-B protocol, it follows by (12) and (14) that

$$\mathbb{P}_{B,i}(n) \doteq p_{sd}\Lambda_{\alpha,\rho}^n + \sum_{k=1}^{n}\sum_{d=1}^{M}\Phi_{d,k}\Lambda_{\alpha,\rho}^{n-k+1}\left(\frac{p_{rd}}{p_{sr}}\right)^d p_{rd}^{d(k-1)} \tag{34}$$

where $\Phi_{d,k} \triangleq \frac{C_d^M}{[i_d!i_d^{(d-i_d)}]^k}$. Following a procedure similar to the proof of Proposition 4 in Appendix D, (34) can also be simplified in different system operating conditions as follows.

Corollary 2: Suppose $p_{sr} < p_{sd} < 1$ and $p_{rd} < p_{sd} < 1$, namely both α and $\beta > 1$, the asymptotic function of $\mathbb{P}_{B,i}(n)$ of the Type-B protocol with ODSTCi relaying is given by

$$\mathbb{P}_{B,i}(n) \cong \begin{cases} p_{sd}^{n+1}p_{sr}^{Mn}, & p_{sr} > \bar{p}_{rd} \\ \Phi_{M,n}p_{sd}p_{rd}^{Mn}, & p_{rd} \ge \hat{p}_{sr} \\ Mp_{sd}p_{sr}^{M-1}p_{rd}^n, & \text{otherwise} \end{cases} \tag{35}$$

where $\bar{p}_{rd} \triangleq \max\{p_{rd}, p_{A_2}\}$, and $\hat{p}_{sr} \triangleq \max\{p_{sr}^{\frac{1}{n}}, p_{B_1}\} \ge \bar{p}_{sr}$ with $p_{B_1} = [\frac{\Phi_{M,n}}{M}(\frac{\alpha}{\delta_s}\frac{\delta_r}{\beta})^{M-1}]^{\frac{-1}{(n-1)(M-1)}} > p_{A_1}$.

The proof for this corollary is omitted as it routinely follows the same procedure in Appendix D for the Type-A protocol. And the diversity of the Type-B protocol is also $\xi_B = M + n$ as shown in (35), attributed to the event of $\mathcal{D} = 1$. The difference between (35) and (32) of the Type-A protocol lies only in the case of $p_{sr} \le p_{rd}^n$. When $p_{rd} \ge p_{B_1}$ as well, i.e., $\rho \le \frac{\delta_r}{\beta} p_{B_1}^{-1}$, the asymptotic function of $\mathbb{P}_{B,i}(n)$ is proportional to $\frac{1}{\rho^{Mn+1}}$ due to the reselection function in every ARQ when $\mathcal{D} \ge 1$.

C. Asymptotic Outage Probability of the Type-C Protocol

Even though the form of $\mathbb{P}_{C,i}(n)$ in (28) is a bit more complicated than those of $\mathbb{P}_{A,i}(n)$ and $\mathbb{P}_{B,i}(n)$, its diversity can be analyzed in a way similar to that for the Type-A and B protocols except that it requires the additional high-SNR approximations of $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|\underline{d}_\ell)$ and $P_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1})$. By Proposition 1, it follows that

$$P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|\underline{d}_\ell) \doteq \frac{\delta_r^{\underline{d}_\ell}}{i_{(\ell+1)}^{(\underline{d}_\ell - i_{\ell+1})} i_{(\ell+1)}!(\beta\rho)^{\underline{d}_\ell}} \tag{36}$$

with $i_\ell \triangleq \min\{i, \underline{d}_{\ell-1}\}$. As for $\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1})$ of (27), it requires the high-SNR approximation of $P_{\mathcal{V}|\mathcal{D}}(\delta_r|d)$ in (26) for relay's overhearing, which, by (30), follows as

$$P_{\mathcal{V}|\mathcal{D}}(\delta_r|i_\ell) \doteq \frac{\delta_r^{i_\ell}}{d!(\eta\rho)^{i_\ell}}. \tag{37}$$

This expression clearly shows that there is no opportunistic diversity being provided to a relay that decodes the DSTC signal sent from other relays. Applying (37) into (27) yields

$$\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1}) \doteq C_{d_\ell}^{M-\underline{d}_{\ell-1}} \left[\frac{\delta_r^{i_\ell}}{i_\ell!(\eta\rho)^{i_\ell}}\right]^{M-\underline{d}_\ell}. \tag{38}$$

Substitute the results back into (28) and define $p_{rr} \triangleq \frac{\delta_r}{\eta\rho} < 1$. The asymptotic function of $\mathbb{P}_{C,i}(n)$ at high SNR is given by

$$\mathbb{P}_{C,i}(n) \doteq p_{sd}\Lambda_{\alpha,\rho}^n + \sum_{k=1}^{n}\sum_{d_0=1}^{M}\sum_{d_1=0}^{M-\underline{d}_0}\cdots\sum_{d_{k-1}=0}^{M-\underline{d}_{k-2}} \times \Psi_{i,k}\Lambda_{\alpha,\rho}^{n-k+1}\left(\frac{p_{rd}}{p_{sr}}\right)^{d_0}\prod_{\ell=1}^{k-1} p_{rd}^{d_\ell} p_{rr}^{i_\ell(M-\underline{d}_\ell)} \tag{39}$$

where $\Psi_{i,k} \triangleq \frac{C_{d_0}^M}{i_1^{(\underline{d}_0 - i_1)} i_1!}\prod_{\ell=1}^{k-1}\frac{C_{d_\ell}^{M-\underline{d}_{\ell-1}}}{i_{(\ell+1)}^{(\underline{d}_\ell - i_{\ell+1})} i_{(\ell+1)}!(i_\ell!)^{M-\underline{d}_\ell}}$.

Clearly, the second term on the RHS of (39) is dominated at high SNR by the feasible pairs of $(\underline{d}_\ell, i_\ell), \ell = 1,\ldots,k-1$, $k \ge 2$, that minimize $\sum_{\ell=1}^{k-1}[\underline{d}_\ell + i_\ell(M-\underline{d}_\ell)]$. Since $i_\ell \ge 1$ and $\underline{d}_\ell \le M$, we have $\underline{d}_\ell + i_\ell(M-\underline{d}_\ell) \ge M$. The equality holds if $i = 1$. For $i \ge 2$, we define the set of the increasing number of relays that satisfies the lower bound in k rounds of retransmissions as

$$\mathcal{U}_{k-1} \triangleq \left\{(d_0,\ldots,d_{k-1}) \left| \begin{array}{c} \underline{d}_\ell + i_\ell(M-\underline{d}_\ell) = M \\ d_\ell \in [0, M-\underline{d}_{\ell-1}] \\ \text{and } \forall \ell \in [1, k-1] \end{array}\right.\right\} \tag{40}$$

for $k \ge 2$, $\underline{d}_0 \equiv d_0 \ge 1$, and $i \ge 2$. The algorithm to construct $\mathcal{U}_{k-1}$ is provided in the following proposition.

Proposition 5: Assume $\underline{d}_0 \equiv d_0 \ge 1$ and $i \ge 2$.

1) Let $\mathcal{U}_0 \triangleq \{d_0 | d_0 \in [1, M]\}$.
2) For $\ell = 1$ to $k-1$, $k \ge 2$.

TABLE I
ORDERS OF ρ^{-1} OF THE ASYMPTOTIC OUTAGE PROBABILITIES OF THE THREE ARQ PROTOCOLS IN DIFFERENT OPERATING CONDITIONS

	Typical channels	$p_{rd} > \hat{p}_{sr}$	$p_{sr} > \hat{p}_{rd}$
ξ_A	$M+n$	$1+M+i(n-1)$	$n(M+1)+1$
ξ_B	$M+n$	$1+Mn$	$n(M+1)+1$
ξ_C	$1+Mn$	$1+Mn$	$n(M+1)+1$

Generate $\mathcal{U}_\ell^a \triangleq \{(d_\ell,\ldots,d_0)|d_\ell \in [0, M-2], d_0 = 1 \text{ and } d_1 = \cdots = d_{\ell-1} = 0 \text{ if } \ell \ge 2\}$ and $\mathcal{U}_\ell^b \triangleq \{(d_\ell,\ldots,d_0)|d_\ell = M - \underline{d}_{\ell-1}, (d_{\ell-1},\ldots,d_0) \in \mathcal{U}_{\ell-1}\}$, and set $\mathcal{U}_\ell \triangleq \mathcal{U}_\ell^a \cup \mathcal{U}_\ell^b$.

Proof: The proof is provided in Appendix E. ∎

Given $\mathcal{U}_{k-1}$ and, thus, $i_\ell(M-\underline{d}_\ell) = M - \underline{d}_\ell \ \forall \ell \in [1, k-1]$, the second term on the RHS of (39) can be obtained as

$$\sum_{k=1}^{n}\sum_{\mathcal{U}_{k-1}}\Psi_{i,k}\Lambda_{\alpha,\rho}^{n-k+1}\left(\frac{p_{rd}}{p_{sr}}\right)^{d_0}\prod_{\ell=1}^{k-1} p_{rd}^{d_\ell} p_{rr}^{M-\underline{d}_\ell}. \tag{41}$$

Since the order of ρ in (41) equals to $(M+1)(n-k+1)+(k-1)M$ which decreases when k increases, keeping only the dominant term that corresponds to $k = n$, we obtain

$$\mathbb{P}_{C,i}(n) \doteq p_{sd}\Lambda_{\alpha,\rho}^n + \sum_{\mathcal{U}_{n-1}}\Psi_{i,n}\Lambda_{\alpha,\rho}\left(\frac{p_{rd}}{p_{sr}}\right)^{d_0}\prod_{\ell=1}^{n-1} p_{rd}^{d_\ell} p_{rr}^{M-\underline{d}_\ell}. \tag{42}$$

The diversity order of (42) is equal to $\xi_C = Mn+1$ as $\rho \to \infty$. For finite SNR, however, if the first term on the RHS of (42) is greater than the second term, namely when $p_{sr} > p_C$, or equivalently $\rho < \frac{\delta_s}{\alpha} p_C^{-1}$, with p_C being the value of p_{sr} that makes the first term equal to the second term on the RHS of (42), then the outage events become mainly attributed to the transmission failures between the source and relays. The Type-C protocol will perform exactly the same as the Type-A and -B protocols in this case, i.e., $\mathbb{P}_{C,i}(n) \doteq p_{sd}^{n+1} p_{sr}^{Mn}$ for $p_{sr} > p_C$. Under this circumstance, there will be no need for the Type-C protocol at all. Therefore, if we define $\hat{p}_{rd} \triangleq \max\{p_C, \tilde{p}_{rd}\}$, and abuse the diversity notation ξ to represent the dominant orders of ρ^{-1} of the asymptotic function in finite SNR regimes, then by (32), (33), (35), and the aforementioned results, we may summarize ξ of the three ARQ protocols in different operating conditions as in Table I.

D. Relay Efficiencies of ODSTCi in the Three ARQ Protocols

Based on the above asymptotic analysis, we continue to study the relay efficiencies of ODSTCi in the three ARQ protocols. Due to the complexity of analysis, closed-form expressions are obtained for the Type-A and B protocols only.

Recall from (32) and (33) that in typical operating conditions, i.e., when $p_{sr} < \bar{p}_{rd}$ or $p_{rd} < \bar{p}_{sr}$, the diversity of $\mathbb{P}_{A,i}(n)$ is $\xi_A = M+n$. We may, thus, define a coding gain

$$G_A(i,n) \triangleq \lim_{\rho\to\infty}\frac{\mathbb{P}_{A,i}(n)}{\rho^{-(M+n)}} \doteq \begin{cases} \frac{\Delta_{\alpha,\beta}\delta_s^M\delta_r^n}{\alpha^{(M-1)}\beta^n}, & i = 1 \\ \frac{M\delta_s^M\delta_r^n}{\alpha^{(M-1)}\beta^n}, & i \ne 1 \end{cases} \tag{43}$$

to quantify the SNR gain per active relays for ODSTCi in this type of ARQ. The results, however, are negative except for the case of $i \leq 2$. The SNR gain $G_A(i,n) - G_A(i-1,n)$ is zero when $3 \leq i \leq M$. On the other hand, we may also follow a definition similar to (19) in Section III-A to quantify the SNR losses of using ODSTCi than using DSTC, which leads to

$$\mathcal{L}_A(1,n) = \frac{1}{M+n} \log\left\{\frac{G_A(1,n)}{G_A(M,n)}\right\} = \frac{1}{M+n} \log\left\{\frac{\Delta_{\alpha,\beta}}{M}\right\} \tag{44}$$

and $\mathcal{L}_A(i,n) = 0$, for $i \neq 1$. A similar result of $\mathcal{L}_B(i,n) = 0$ holds for the Type-B protocol $\forall i \in \{1, M\}$ as shown in (35). However, if we extend the definition of coding gain to the asymptotic outage probabilities, then, by (35), we can have $G_B(i,n) \triangleq \frac{\Phi_{M,n} p_{sd} p_{rd}^{Mn}}{\rho^{-(Mn+1)}} = \frac{\delta_s \delta_r^{Mn}}{\beta^{Mn}(i! i^{M-i})^n}$ when $p_{rd} \geq \hat{p}_{sr}$, i.e., $\rho \leq \frac{\delta_r}{\beta}\hat{p}_{sr}^{-1}$. Consequently, by the definition of (19), we obtain

$$\mathcal{L}_B(i,n) = \begin{cases} \frac{n}{1+nM}\log\left\{\frac{M!}{i! i^{M-i}}\right\}, & p_{rd} \geq \hat{p}_{sr} \\ 0, & \text{otherwise.}\end{cases} \tag{45}$$

This shows that when the performance is mainly limited by the R-D link quality, then using more active relays for ODSTCi does introduce an SNR gain, even though it also decreases rapidly with i. Otherwise, there is no need to use for more than one relay for the Type-B protocol. In such kind of channels, the Type-B protocol, in fact, performs exactly the same as the Type-A protocol at high SNR when $i \geq 2$ since $\mathcal{L}_A(i,n) = 0\ \forall i \geq 2$, and $\mathbb{P}_{A,M}(n) = \mathbb{P}_{B,M}(n)$. Only the Type-C protocol can attain the full diversity in such channels, whose SNR gain $G_C(i,n) - G_C(i-1,n)$ and $\mathcal{L}_C(i,n)$ can be evaluated numerically by (42) and the definition of (19).

VI. Throughput Analysis and Protocol Effectiveness

The previous analysis allows us to quantify the diversity enhancement for every retransmission and the relay efficiencies, in terms of the SNR gain per active relay for ODSTCi, in the three protocols. In practice, we would also like to know the advantages of the different schemes in different channel conditions, after all the Type-C protocol has a higher complexity than the other two. We investigate the effectiveness of the increased levels of complexities for relays overhearing and reselection from the system deployment and throughput points of view. We start with the throughput analysis.

A. Throughput Analysis

Define $\overline{\mathbb{P}}_{\mathcal{T},i}(n_s, n_r)$ as the probability of successful packet deliveries after n_s rounds of retransmissions with source broadcasting followed by n_r rounds of retransmissions with ODSTCi relaying in the type $\mathcal{T} \in \{A,B,C\}$ protocol. The average throughput in a total of n rounds of retransmissions can be formulated as

$$\zeta_{\mathcal{T},i}(n) \triangleq \sum_{n_s=0}^{n} \sum_{n_r=0}^{n-n_s} \frac{R_s \cdot \overline{\mathbb{P}}_{\mathcal{T},i}(n_s,n_r)}{1+n_s+n_r R_r/R_s} \tag{46}$$

where the probabilities $\overline{\mathbb{P}}_{\mathcal{T},i}(n_s,n_r)$ of the three types of ARQ protocols are provided in the following proposition.

Proposition 6: Given R_s, R_r, M, and (n_s, n_r), the probabilities $\overline{\mathbb{P}}_{\mathcal{T},i}(n_s,n_r)$ for $\mathcal{T} \in \{A,B,C\}$ are given by

1) If $n_r = 0$, then

$$\overline{\mathbb{P}}_{\mathcal{T},i}(n_s,0) = [P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)]^{n_s}[1 - P_W(\delta_s)]. \tag{47}$$

2) If $n_r = 1$, then

$$\overline{\mathbb{P}}_{\mathcal{T},i}(n_s,1) = P_W^{n_s+1}(\delta_s)\mathcal{P}_{\mathcal{D}}^{n_s}(0)\sum_{d=1}^{M}\mathcal{P}_{\mathcal{D}}(d) \times [1 - P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)]. \tag{48}$$

3) For $n_r \geq 2$, we have

$$\overline{\mathbb{P}}_{A,i}(n_s,n_r) \triangleq P_W^{n_s+1}(\delta_s)\mathcal{P}_{\mathcal{D}}^{n_s}(0)\sum_{d=1}^{M}\mathcal{P}_{\mathcal{D}}(d)P_{\mathcal{O}_i|\mathcal{D}} \times (\delta_r|d)P_{D_i|\mathcal{D}}^{n_r-2}(\delta_r|d)[1 - P_{D_i|\mathcal{D}}(\delta_r|d)] \tag{49}$$

$$\overline{\mathbb{P}}_{B,i}(n_s,n_r) \triangleq P_W^{n_s+1}(\delta_s)\mathcal{P}_{\mathcal{D}}^{n_s}(0)\sum_{d=1}^{M}\mathcal{P}_{\mathcal{D}}(d) \times P_{\mathcal{O}_i|\mathcal{D}}^{n_r-1}(\delta_r|d)[1 - P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)] \tag{50}$$

and

$$\begin{aligned}\overline{\mathbb{P}}_{C,i}(n_s,n_r) \triangleq{}& P_W^{n_s+1}(\delta_s)\mathcal{P}_{\mathcal{D}_0}^{n_s}(0)\sum_{d_0=1}^{M}\sum_{d_1=0}^{M-d_0}\cdots\sum_{d_{n_r-1}=0}^{M-\underline{d}_{n_r-2}} \\ &\times \left\{\prod_{\ell=1}^{n_r-2} P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|\underline{d}_\ell)\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1})\right\} \\ &\times \mathcal{P}_{\mathcal{D}_0}(d_0)P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d_0)\mathcal{P}_{\mathcal{D}_{n_r-1}|\underline{\mathcal{D}}_{n_r-2}} \\ &\times (d_{n_r-1}|\underline{d}_{n_r-2})[1 - P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|\underline{d}_{n_r-1})].\end{aligned} \tag{51}$$

Proof: The proof can be readily obtained by following the proofs of the outage probabilities of $\mathbb{P}_{\mathcal{T},i}(n)$, $\mathcal{T} \in \{A,B,C\}$, in Section IV. Thus, only a sketch of the proof is provided.

If $n_r = 0$, (47) represents the probability that $\mathcal{S}_D$ remains empty and the destination successfully decodes the packet at the n_sth rebroadcasting from the source. If $n_r \geq 1$, it implies that the packet is delivered by the relays. Take the Type-A protocol as an example. The second term on the RHS of (22) represents the probability that $\mathcal{S}_D$ turns nonempty at the $(n-k)$th rebroadcasting and that the packet still cannot be successfully delivered after the following k rounds of relay forwarding. Equations (48) and (49) can be obtained by replacing the last product term by $1 - P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)$ and $1 - P_{D_i|\mathcal{D}}(\delta_r|d)$, respectively. For the Type-B and Type-C protocols, (50) and (51) can also be obtained by following the same approach. ■

B. Numerical Study on the Effectiveness of Different Protocols

Based on our analysis, we examine by simulations the effectiveness and relay efficiencies of the three protocols in different channel conditions. Fig. 6 shows the results of $\mathbb{P}_{A,i}(2)$, $\mathbb{P}_{A,i}(3)$ of (22), and $\mathbb{P}_{C,i}(3)$ of (28), and their asymptotic functions (dotted lines) in (32), (33), and (39) when $M = 4$, $\alpha = 32$,

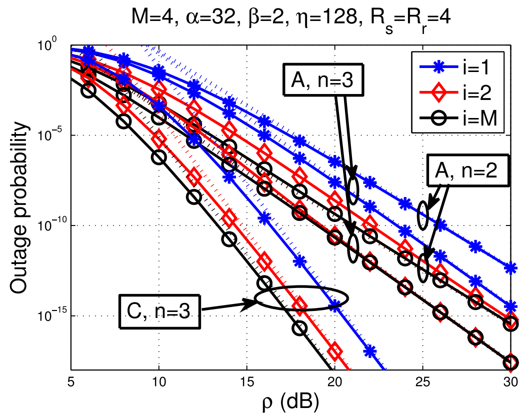

Fig. 6. Outage probabilities and their asymptotic functions of the Type-A and -C protocols when $M = 4$, $\alpha = 32$, $\beta = 2$, $\eta = 128$, and $R_s = R_r = 4$.

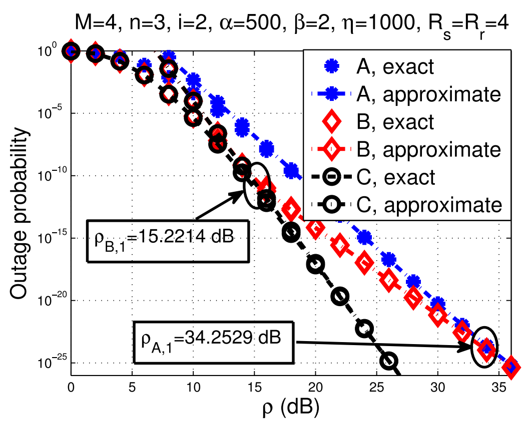

Fig. 7. Outage probabilities of the Type-A, -B, and -C protocols.

$\beta = 2$, $\eta = 128$, and $R_s = R_r = 4$ (b/s/cu). To verify the relay efficiency of ODSTCi for the Type-A protocol, we consider two cases for $i = 1$ and 2, and compare the results with the ordinary DSTC that uses all available relays in $\mathcal{S}_D$. Clearly, the asymptotic functions match the outage probabilities at high SNR. Besides, consistent with $\mathcal{L}_A(i, n)$ of (44), the simulated results also show that the SNR losses for using ODSTCi than DSTC for the Type-A protocol are zero at high SNR for $i \geq 2$.

Compared to the Type-A protocol, the Type-C has considerable advantage in this case given that its diversity order achieves $1 + Mn$ while the diversity orders of the Type-A and B are both $M + n$ according to Table I Besides, the SNR gain is zero for the Type-A protocol when $i \geq 3$, while we may use ODSTC3 to obtain an additional SNR gain for the Type-C since its SNR loss $\mathcal{L}_C(2, 3)$ is still around 1 dB. Given that its outage probability is no longer limited by the event of $\mathcal{D}_0 = 1$, using more relays, thus, offers some power gains.

In a more radical case, we consider $\alpha = 500$, $\beta = 2$, and $\eta = 10^3$, for simulations in Fig. 7. For a power path loss proportional to the cube of the propagation distance, this setting can be associated with a scenario where the R-D distance is 6.3 times longer than the S-R distance, and 7.9 times longer than the R-R distance. For $\delta_s = \delta_r = 15, n = 3$, and $M = 4$, by (32), we have

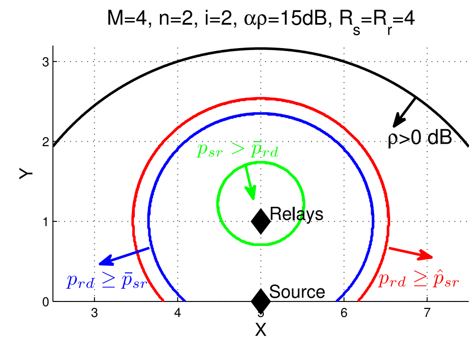

Fig. 8. Network view on the different operating conditions in Table I.

$p_{rd} = \frac{\delta_r}{\beta\rho} = \frac{15}{2\rho} > p_{A_1} = 2.82 \times 10^{-3}$ if $\rho < \rho_{A,1} \triangleq 34.25$ dB. If $p_{rd} = \frac{15}{2\rho} \geq p_{sr}^{\frac{1}{3}} = (\frac{15}{500\rho})^{\frac{1}{3}}$ as well, i.e., $\rho \leq$20.74 dB, then $p_{rd} \geq \bar{p}_{sr}$. According to (32) and Table I, ξ_A of the Type-A protocol is $M + 1 + i(n - 1) = 9$, even though it will eventually converge to $M + n = 7$ when $\rho \geq \rho_{A,1}$.

In comparison, $\mathbb{P}_{B,2}(3)$ of the Type-B protocol follows the trend of $\mathbb{P}_{C,2}(3)$ of the Type-C protocol, whose diversity order is $\xi_C = Mn + 1 = 13$, when $\rho < \rho_{B,1} \triangleq 15.22$ dB such that $p_{rd} > p_{B_1}$ and, hence, $p_{rd} \geq \hat{p}_{sr}$ by (35). While $\mathbb{P}_{B,2}(3)$ diverts away from $\mathbb{P}_{C,2}(3)$ when $\rho > \rho_{B,1}$ and converges to $\mathbb{P}_{A,2}(3)$ of the Type-A when $\rho > \rho_{A,1}$, it ends up with a diversity order of $\xi_B = M + n = 7$. It is noted that $\mathcal{L}_A(i, n) = \mathcal{L}_B(i, n) = 0$ for $i \geq 2$, and $\mathbb{P}_{A,M}(n) = \mathbb{P}_{B,M}(n)$.

In Fig. 8, we study from a system deployment viewpoint the effectiveness of the reselection and the overhearing functions in the Type-B and -C protocols. To this end, we fix the S-R SNR $\alpha\rho$ at 15 dB and examine the diversities of the three protocols for $n = 2$ at different locations in the coverage range of the network defined as $\rho \geq$0 dB, i.e., the black curve in the plot. Other simulation settings are the same to those in Fig. 7.

When the destination is inside the green circle, we have $p_{sr} > \bar{p}_{rd}$ such that the three protocols have the same diversity of $n(M + 1) + 1$. Thus, there is no need for the Type-B and -C. While if the destination is outside the red circle, we have $p_{rd} > \hat{p}_{sr}$. By (35), the Type-B reaches the full diversity such that the overhearing function of the Type-C is unnecessary. As for the Type-A protocol, it has the diversity of $M + 1 + i(n - 1)$ when $p_{rd} > \bar{p}_{sr}$, namely, the destination is outside the blue circle. The Type-C is more effective in diversity only in the typical operating range between the green and the blue circles.

As for the effectiveness of the two functions in throughput enhancement, the simulations for $M = 5$, $\beta = 2$, $\eta = 64$, and $n = 2$ are presented in Fig. 9. The data rates R_s and R_r are adjusted w.r.t. the SNR to maximize the throughput $\zeta_{\mathcal{T},i}(2)$ of (46) subject to $\mathbb{P}_{\mathcal{T},i}(2) \leq 10^{-3}, \mathcal{T} \in \{A, B, C\}$. Intuitively, the Type-C protocol will offer the best throughput since its overhearing function can resolve the relay shortage issue in $\mathcal{S}_D$ and achieve the highest diversity, especially when the S-R link quality is slightly worse or comparable to the R-D link quality as in the case of $\alpha = 2$ in Fig. 9. Under this operating condition,

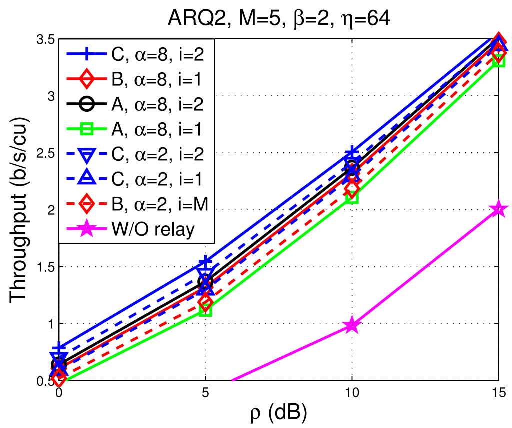

Fig. 9. Throughput of the three protocols when $\mathbb{P}_{\mathcal{T},i}(2) \leq 10^{-3}$.

we can see that $\zeta_{B,M}(2)$ of the Type-B protocol is about 0.3 (b/s/cu) inferior to $\zeta_{C,2}(2)$ of the Type-C.

On the contrary, when the S-R link quality becomes significantly better as in the case of $\alpha = 8$ in Fig. 9, $\zeta_{B,1}(2)$ of the Type-B protocol with OR or $\zeta_{A,2}(2)$ of the Type-A with ODSTC2 are very close and provide satisfactory throughput in comparison with $\zeta_{C,2}(2)$ of the Type-C. These observations agree with our diversity analysis in (44) and (45), and the simulations in Fig. 8, and together provide us an insight into the choice of a proper ARQ protocol in different operation conditions. In summary, when the R-D link quality is comparable to that of the S-R link, the Type-C protocol with ODSTC2 can provide a significant performance enhancement. However, when the R-D link becomes notably worse, either the Type-B that reselects a single relay for OR in each retransmission, or the Type-A that needs no reselection but rather synchronization among relays to use ODSTC2 for ARQ is a good performance tradeoff for practical implementations.

VII. Conclusion

We studied the relay efficiencies and protocol effectiveness of using ODSTCi for relay-assisted ARQ. Three types of protocols were introduced in this regard, allowing us to examine the effectiveness of the increased levels of complexities for diversity enhancement in the three protocols. Starting with the analysis for the capacity outage probability of using ODSTCi relaying in Rayleigh fading channels, we gradually derived the outage probabilities of the three protocols and successfully characterized their relay efficiencies in using ODSTCi.

Through our analysis and simulation studies, we also found that the effectiveness and relay efficiencies of the three protocols are closely related to the relative data rate settings and channel qualities on the S-R and R-D links. In certain operating conditions, the Type-B protocol can be as effective as the Type-C, while the efficiencies of using more than two active relays for ODSTCi are usually negligible in all three protocols. Specifically speaking, in a typical operating condition where the S-R link quality is slightly worse or comparable to the R-D link quality, using only ODSTC2 for the Type-C protocol can effectively enlarge the relay set $\mathcal{S}_D$ and offer significant throughput enhancement. On the contrary, when the R-D link quality becomes significantly worse, then using the Type-B protocol can attain the full diversity, and from a throughput point of view, either the Type-A with ODSTC2 or the Type-B with OR provide a satisfactory throughput.

Appendix A
Capacity Outage Probability of ODSTCi

The achievable capacity of OMPR with DSTC can actually be represented as a function of the sum of the top order statistics. The capacity outage probability w.r.t. a predefined threshold can thus be derived based on [30, Th. 11.3.1]. Given $X_j \sim \text{Exp}(\lambda_{2,j})$, $j \in \mathcal{S}_D$, we define $\lambda_{2,a} \neq \lambda_{2,b}$ if $a \neq b$. Conditioned on $|\mathcal{S}_D| \geq 1$, the outage probability of OMPR with DSTC for the case of $i = 1$ is given by

$$\Pr\{\max\{X_j \mid j \in \mathcal{S}_D\} < x\} = \prod_{j \in \mathcal{S}_D} \left(1 - e^{-\lambda_{2,j} x}\right). \tag{52}$$

If $i \geq d$, we define $X_{\text{sum}} \triangleq \sum_{j \in \mathcal{S}_D} X_j$. Since every X_j is independent, the moment generating function (MGF) of X_{sum} is equal to the product of the MGF of each X_j, $j \in \mathcal{S}_D$

$$\mathcal{M}_{X_{\text{sum}}}(t) = \prod_{j \in \mathcal{S}_D} \left(1 - \frac{t}{\lambda_{2,j}}\right)^{-1} = \sum_{j \in \mathcal{S}_D} \frac{\Theta_{j,\mathcal{D}}}{1 - t/\lambda_{2,j}} \tag{53}$$

where $\Theta_{j,\mathcal{D}} = \prod_{\ell \in \mathcal{S}_D, \ell \neq j} (1 - \frac{\lambda_{2,j}}{\lambda_{2,\ell}})^{-1}$. Since (53) is a simple summation form, its inverse Fourier transform, i.e., the PDF of X_{sum}, can be readily obtained. Then, the cumulative distribution function (CDF) of X_{sum} can thus be given by

$$F_{X_{\text{sum}}}(x) = \Pr\{X_{\text{sum}} < x\} = 1 - \sum_{j \in \mathcal{S}_D} \Theta_{j,\mathcal{D}} e^{-\lambda_{2,j} x}. \tag{54}$$

When $1 < i < d$, we recall that $\mathcal{X}'$ represents a set whose elements are those of $\mathcal{X}$ but arranged in the ascending order, and hence, there are $d!$ possible arrangements. As a result, the joint PDF of $X'_1, \ldots, X'_{\mathcal{D}}$ is the sum of $d!$ terms, and each term is given by

$$\prod_{k=1}^{d} \lambda_{2,r(k)} e^{-\lambda_{2,r(k)} x_k} \, dx_k, \; 0 < x_1 < \cdots < x_d < \infty \tag{55}$$

where $(r(1), \ldots, r(d))$ represents a sequence of numbers that denote the permutation of the index set $\{j | j \in \mathcal{S}_D\}$, yielding $X_{r(k)} = X'_k$, $k = 1, \ldots, d$. Thus, the complimentary capacity outage probability of ODSTCi can be formulated as

$$\Pr\left\{ \sum_{j=d-i+1}^{d} X_j{}' > \delta_r \,\middle|\, d \geq 1 \right\}$$
$$= \sum_{d!} \underset{\substack{0 < x_1 < x_2 < \cdots < x_d < \infty \\ x_{d-i+1} + \cdots + x_d > \delta_r}}{\int \cdots \int} \prod_{k=1}^{d} \lambda_{2,r(k)} e^{-\lambda_{2,r(k)} x_k} \, dx_k. \tag{56}$$

Following the same approach in [30], we split and group the $d!$ terms into the following cases:

1) Let $X'_{d-i} = X_{r(d-i)} = X_j = x$ with $\lambda_{2,j} = \lambda_{2,r(d-i)}$.
2) Given $j \in \mathcal{S}_D$, we define $\mathcal{S}_L(j) \triangleq \{m | m \in \mathcal{S}_D,\ X_m \in \{X'_1, \ldots, X'_{d-i-1}\},\ m \neq j\}$ to identify the relays which are in $\mathcal{S}_D$ and have lower channel strength than X_j. In addition, we further define $\mathcal{S}'_L(j)$ as a typical ordered set of $\mathcal{S}_L(j)$, and there are $C^{d-1}_{d-i-1}(d-i-1)!$ possible combinations of $\mathcal{S}'_L(j)$.
3) The remaining relays, which have stronger channel strength than X_j, compose the set $\mathcal{S}_U(j) \triangleq \{m | m \in \mathcal{S}_D,\ \ X_m \in \{X'_{d-i+1}, \ldots, X'_d\},\ \ m \neq j\}$ and denote $\mathcal{S}'_U(j)$ as a typical ordered set of $\mathcal{S}_U(j)$.

Accordingly, (56) can be simplified as

$$\sum_{j=1}^{d} \int_0^\infty \lambda_{2,j} e^{-\lambda_{2,j} x} \underbrace{\sum_{\mathcal{S}'_L(j)} \int\cdots\int_{0<x_1<\cdots<x_{d-i-1}<x} \prod_{k=1}^{d-i-1} \frac{\lambda_{2,r(k)}}{e^{\lambda_{2,r(k)} x_k}} dx_k}_{(a)}$$

$$\times \underbrace{\sum_{\mathcal{S}'_U(j)} \int\cdots\int_{\substack{0<x<x_{d-i+1}<\cdots<x_d<\infty \\ x_{d-i+1}+\cdots+x_d>\delta_r}} \prod_{k=d-i+1}^{d} \frac{\lambda_{2,r(k)}}{e^{\lambda_{2,r(k)} x_k}} dx_k dx}_{(b)}. \quad (57)$$

Actually, (a) of (57) is the probability that all the elements in $S(j)$ are less than x, and can, thus, be given by (52), which follows:

$$\begin{aligned}(a) &= \sum_{\mathcal{S}_L(j)} \Pr\left\{\max_{k\in\mathcal{S}_L(j)} X_k < x\right\} \\ &= \sum_{\mathcal{S}_L(j)} \prod_{k\in\mathcal{S}_L(j)} \left(1-e^{-\lambda_{2,k} x}\right). \end{aligned} \quad (58)$$

Based on (54), let $Y_k = X_k - x, k = d-i+1, \ldots, d$. We may simplify (b) of (57) as

$$\begin{aligned}(b) &= e^{-x\sum_{q\in\mathcal{S}_U(j)} \lambda_{2,q}} \Pr\left\{\sum_{k\in\mathcal{S}_U(j)} Y_k > \delta_r - ix\right\} \\ &= e^{-x\sum_{q\in\mathcal{S}_U(j)} \lambda_{2,q}} \sum_{k\in\mathcal{S}_U(j)} \Theta_{k,M}\, e^{-\lambda_{2,k}(\delta_r - ix)}. \end{aligned} \quad (59)$$

Substituting (58) and (59) into (57), the outage probability $P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d) = 1 - \Pr\{\sum_{j=d-i+1}^{d} X'_j > \delta_r | d \geq 1\}$ can finally be expressed as

$$\begin{aligned}1 - \sum_{j=1}^{d} \lambda_{2,j} \sum_{\mathcal{S}_L(j)} \Bigg[\int_{\frac{\delta_r}{i}}^{\infty} \prod_{m\in\mathcal{S}_L(j)} \left(1-e^{-\lambda_{2,m} x}\right) \\ \times e^{-(\lambda_{2,j}+\sum_{q\in\mathcal{S}_U(j)}\lambda_{2,q})x} dx + \sum_{k\in\mathcal{S}_U(j)} \frac{e^{-\lambda_{2,k}\delta_r}}{\prod_{\ell\in\mathcal{S}_U(j),\ell\neq k}\left(1-\frac{\lambda_{2,k}}{\lambda_{2,\ell}}\right)} \\ \times \int_0^{\frac{\delta_r}{i}} \prod_{m\in\mathcal{S}_L(j)} \left(1-e^{-\lambda_{2,m} x}\right) e^{-(\lambda_{2,j}+\sum_{q\in\mathcal{S}_U(j)}\lambda_{2,q} - i\lambda_{2,k})x} dx\Bigg]. \end{aligned} \quad (60)$$

APPENDIX B
PROOF OF PROPOSITION 1

To find the closed-form expression of $\lim_{\lambda\to 0} \frac{P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)}{\lambda^d}$, we would need an asymptotic result of the CDF given below.

Assume $\mathcal{D} \geq i$. Let $\mathcal{O}_i \triangleq \sum_{j=\mathcal{D}-i+1}^{\mathcal{D}} X'_j$ stand for the summation of the largest i out of $\mathcal{D}$ independent identically distributed exponential RVs, $X_j \sim \text{Exp}(\lambda/\beta)$, with $X'_\mathcal{D} \geq X'_{\mathcal{D}-1} \geq, \ldots, X'_1 \geq 0$. For the CDF of $\mathcal{O}_i < \delta_r$, we have

$$\begin{aligned}F_{\mathcal{O}_i}(\delta_r) = \int_0^{\frac{\delta_r}{i}} \int_{x'_{\mathcal{D}-i+1}}^{\frac{\delta_r - x'_{\mathcal{D}-i+1}}{i-1}} \cdots \int_{x'_{\mathcal{D}-1}}^{\delta_r - x'_{\mathcal{D}-i+1} - \ldots - x'_{\mathcal{D}-1}} \\ \times f_{X'_\mathcal{D},\ldots,X'_{\mathcal{D}-i+1}}(x'_\mathcal{D}, \ldots, x'_{\mathcal{D}-i+1}) dx'_\mathcal{D} \cdots dx'_{\mathcal{D}-i+1}\end{aligned} \quad (61)$$

where the joint PDF of the ordered RVs $X'_\mathcal{D}, \ldots, X'_{\mathcal{D}-i+1}$ is shown by [31]

$$\frac{\mathcal{D}!}{(\mathcal{D}-i)!} \frac{\lambda^i}{\beta^i} e^{-\frac{\lambda}{\beta}\sum_{\ell=0}^{i-1} x'_{\mathcal{D}-\ell}} \left(1 - e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-i+1}}\right)^{\mathcal{D}-i}. \quad (62)$$

To derive the expression of $\lim_{\lambda\to 0} \frac{1}{\lambda^\mathcal{D}} F_{\mathcal{O}_i}(\delta_r)$, we compute its upper and lower bounds. For the lower bound, exploiting Fatou's lemma [32], we obtain

$$\begin{aligned}\liminf_{\lambda\to 0} \frac{F_{\mathcal{O}_i}(\delta_r)}{\lambda^\mathcal{D}} \geq \int_0^{\frac{\delta_r}{i}} \cdots \int_{x'_{\mathcal{D}-1}}^{\delta_r - x'_{\mathcal{D}-i+1} - \ldots - x'_{\mathcal{D}-1}} \\ \times \liminf_{\lambda\to 0} \frac{f_{X'_\mathcal{D},\ldots,X'_{\mathcal{D}-i+1}}(x'_\mathcal{D}, \ldots, x'_{\mathcal{D}-i+1})}{\lambda^\mathcal{D}} dx'_\mathcal{D} \cdots dx'_{\mathcal{D}-i+1}\end{aligned} \quad (63)$$

where

$$\begin{aligned}&\liminf_{\lambda\to 0} \frac{f_{X'_\mathcal{D},\ldots,X'_{\mathcal{D}-i+1}}(x'_\mathcal{D}, \ldots, x'_{\mathcal{D}-i+1})}{\lambda^\mathcal{D}} \\ &\quad = \liminf_{\lambda\to 0} \frac{\lambda^i \mathcal{D}! \left(1 - e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-i+1}}\right)^{\mathcal{D}-i}}{\lambda^\mathcal{D} (\mathcal{D}-i)! \beta^i} \prod_{\ell=0}^{i-1} e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-\ell}} \\ &\quad = \frac{\mathcal{D}!}{(\mathcal{D}-i)!} \frac{(x'_{\mathcal{D}-i+1})^{\mathcal{D}-i}}{\beta^\mathcal{D}}\end{aligned} \quad (64)$$

due to the fact that $\lim_{\lambda\to 0} \inf e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-\ell}} = 1$ and $\lim_{\lambda\to 0} \inf \frac{(1-e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-i+1}})^{\mathcal{D}-i}}{\lambda^{\mathcal{D}-i}} = (\frac{x'_{\mathcal{D}-i+1}}{\beta})^{\mathcal{D}-i}$ by L'hôpital's rule. As a result, we have

$$\liminf_{\lambda\to 0} \frac{F_{\mathcal{O}_i}(\delta_r)}{\lambda^\mathcal{D}} \geq \frac{\mathcal{D}!}{(\mathcal{D}-i)! \beta^\mathcal{D}} q(\delta_r, \mathcal{D}, i) \quad (65)$$

where $q(\delta_r, \mathcal{D}, i)$ is defined as

$$\begin{aligned}q(\delta_r, \mathcal{D}, i) \triangleq \int_0^{\frac{\delta_r}{i}} \int_{x'_{\mathcal{D}-i+1}}^{\frac{\delta_r - x'_{\mathcal{D}-i+1}}{i-1}} \cdots \int_{x'_{\mathcal{D}-1}}^{\delta_r - x'_{\mathcal{D}-i+1} - \ldots - x'_{\mathcal{D}-1}} \\ \times (x'_{\mathcal{D}-i+1})^{\mathcal{D}-i} dx'_\mathcal{D} \cdots dx'_{\mathcal{D}-i+1}.\end{aligned} \quad (66)$$

On the other hand, we have $e^{-\frac{\lambda}{\beta}\sum_{\ell=0}^{i-1} x'_{\mathcal{D}-\ell}} \leq 1$ and $1 - e^{-x'_{\mathcal{D}-i+1}/\beta\rho} \leq x'_{\mathcal{D}-i+1}/\beta\rho\ \forall\ x'_{\mathcal{D}-\ell} \geq 0,\ \ell = 0, \ldots, i-1.$

Substituting these upper bounds back into (62), we obtain

$$\lim_{\lambda\to 0}\sup \frac{F_{\mathcal{O}_i}(\delta_r)}{\lambda^{\mathcal{D}}} \le \frac{\mathcal{D}!}{(\mathcal{D}-i)!\beta^{\mathcal{D}}} q(\delta_r, \mathcal{D}, i). \tag{67}$$

Since $\inf(\cdot) \le \sup(\cdot)$, by (65) and (67), we have

$$\lim_{\lambda\to 0} \frac{F_{\mathcal{O}_i}(\delta_r)}{\lambda^{\mathcal{D}}} = \frac{\mathcal{D}!}{(\mathcal{D}-i)!\beta^{\mathcal{D}}} q(\delta_r, \mathcal{D}, i). \tag{68}$$

To further process $q(\delta, \mathcal{D}, i)$, we define a change of variable $T_i = X_{\mathcal{D}-i+1}$, $T_{i-1} = X_{\mathcal{D}-i+2} - X_{\mathcal{D}-i+1}, \cdots, T_1 = X_{\mathcal{D}} - X_{\mathcal{D}-1}$, or equivalently, $X'_j \triangleq \sum_{p=\mathcal{D}-j+1}^{i} T_p$, $j = (\mathcal{D}-i+1), \ldots, \mathcal{D}$. By the change of variable, we have $0 \le T_i = X'_{\mathcal{D}-i+1} \le \frac{1}{i}\sum_{j=\mathcal{D}-i+1}^{\mathcal{D}} X'_j = \frac{\mathcal{O}_i}{i} < \frac{\delta_r}{i}$, and $0 \le T_k < \frac{\delta_r - \sum_{j=k+1}^{i} jT_j}{k}$, $k = 1, \ldots, i-1$. Then, it can be shown as

$$q(\delta_r, \mathcal{D}, i) = \int_0^{\frac{\delta_r}{i}} \int_0^{\frac{\delta_r - it_i}{i-1}} \cdots \int_0^{\delta_r - \sum_{j=2}^{i} j\cdot t_j} t_i^{\mathcal{D}-i} dt_1 \cdots dt_i. \tag{69}$$

This multiple integrals can be solved with Lemma 1 provided in Appendix C which gives $q(\delta_r, \mathcal{D}, i) = \frac{\delta_r^{\mathcal{D}}(\mathcal{D}-i)!}{i! i^{\mathcal{D}-i}\mathcal{D}!}$. Substituting this expression back into (68), we obtain

$$\lim_{\lambda\to 0} \frac{F_{\mathcal{O}_i}(\delta_r)}{\lambda^{\mathcal{D}}} = \frac{\delta_r^{\mathcal{D}}}{i! i^{\mathcal{D}-i}\beta^{\mathcal{D}}}. \tag{70}$$

Applying this formula to $\lim_{\lambda\to 0}\frac{1}{\lambda^d} P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)$ gives

$$\lim_{\lambda\to 0} \frac{1}{\lambda^d} P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d) = \frac{\delta_r^d}{i! i^{d-i}\beta^d},\ d \ge i. \tag{71}$$

In cases where $d < i$, then all the relays in $\mathcal{S}_D$ will be used to forward the signal. The corresponding expression of $\lim_{\lambda\to 0}\frac{1}{\lambda^d} P_{\mathcal{O}_i|\mathcal{D}}(\delta_r|d)$ is equal to setting $i = d$ in (71). Define $i_d \triangleq \min\{i, d\}$ thus, we have (13).

Appendix C
Proof of Lemma 1

Lemma 1: When $L \ge n$, L and n are nature numbers, we have

$$\int_0^{\frac{T}{n}} \int_0^{\frac{T-nt_n}{n-1}} \cdots \int_0^{\frac{T-nt_n-\cdots-2t_2}{1}} t_n^{L-n} dt_1 dt_2 \cdots dt_n$$
$$= \frac{T^L(L-n)!}{n! n^{L-n} L!} \triangleq F_n(T, L). \tag{72}$$

Proof: From the table of integrals [33], we have

$$\int_0^u (u-x)^{p-1} x^{v-1} dx = u^{p+v-1}\frac{(p-1)!(v-1)!}{(p+v-1)!}. \tag{73}$$

Let $n = 2$, by (73), we obtain

$$\int_0^{\frac{T}{2}} \int_0^{T-2t_2} t_2^{L-2} dt_1 dt_2 = 2\int_0^{\frac{T}{2}} \left(\frac{T}{2} - t_2\right) t_2^{L-2} dt_2$$
$$= \frac{T^L(L-2)!}{2\cdot 2^{L-2}\cdot L!} = F_2(T, L). \tag{74}$$

With some mathematical manipulations, we can obtain by induction the form of (72), which completes the proof. ■

Appendix D
Proof of Proposition 4

Given $i_d \triangleq \min\{i, d\}$, $p_{sd} \triangleq \frac{\delta_s}{\rho}$, $p_{sr} \triangleq \frac{\delta_s}{\alpha\rho}$, and $p_{rd} \triangleq \frac{\delta_r}{\beta\rho}$ with $0 < p_{sd}, p_{sr}, p_{rd} < 1$, we may define $\Upsilon_k(\alpha, \beta, \rho) \triangleq \sum_{d=1}^{M} \Omega_{d,k}(\frac{p_{rd}}{p_{sr}})^d p_{rd}^{i_d(k-1)}$ to write $\mathbb{P}_{A,i}(n)$ of (31) as

$$\mathbb{P}_{A,i}(n) \doteq p_{sd}\Lambda_{\alpha,\rho}^n + \sum_{k=1}^{n} \Lambda_{\alpha,\rho}^{n-k+1}\Upsilon_k(\alpha, \beta, \rho) \tag{75}$$

where $\Omega_{d,k} \triangleq \frac{C_d^M}{i_d^{(d-i_d)}(i_d!)^k}$ and $0 < \Lambda_{\alpha,\rho} \triangleq p_{sd}p_{sr}^M < 1$. In the sequel, we discuss the dominant terms of $\Upsilon_k(\alpha, \beta, \rho)$ and, hence, (75) under different channel conditions.

1) When $i > 1$ and $1 > p_{sr} > p_{rd}$, it is clear that $\Upsilon_k(\alpha, \beta, \rho)$ is dominated at high SNR by the event of $d = 1$, which results in $\Upsilon_k(\alpha, \beta, \rho) \doteq Mp_{sr}^{-1}p_{rd}^k$ and the second term on the RHS of (75) asymptotically equal to

$$M\frac{p_{rd}}{p_{sr}}\sum_{k=1}^{n}\left(p_{sd}p_{sr}^M\right)^n\left(\frac{p_{rd}}{p_{sd}p_{sr}^M}\right)^{k-1} \doteq Mp_{sd}p_{sr}^{M-1}p_{rd}^n. \tag{76}$$

Compared to the first term on the RHS of (75), which is $p_{sd}^{n+1}p_{sr}^{Mn}$, (76) has a much lower order of ρ^{-1} such that $\mathbb{P}_{A,i}(n) \doteq Mp_{sd}p_{sr}^{M-1}p_{rd}^n$ at high SNR. Nevertheless, when the SNR is not very high, we may have $\mathbb{P}_{A,i}(n) \doteq p_{sd}^{n+1}p_{sr}^{nM} > Mp_{sd}p_{sr}^{M-1}p_{rd}^n$ under certain channel conditions that satisfy $1 > p_{sr}^{M(n-1)+1} > M(\frac{p_{rd}}{p_{sd}})^n$ or, equivalently, $1 > p_{sr} > [\frac{1}{M}(\frac{\delta_s}{\delta_r}\beta)^n]^{\frac{-1}{M(n-1)+1}} \triangleq p_{A,2}$.

2) When $i > 1$ and $p_{rd} \ge p_{sr}$, in typical channels, we have $1 > p_{rd} \ge p_{sr} > p_{rd}^k$, namely $1 > \frac{\delta_r}{\beta\rho} \ge \frac{\delta_s}{\alpha\rho} > (\frac{\delta_r}{\beta\rho})^k$ for $n \ge k > k'$ with $k' \triangleq \lfloor\frac{\log(\alpha/\delta_s)+\log\rho}{\log(\beta/\delta_r)+\log\rho}\rfloor \ge 1$. By the definition, $\Upsilon_k(\alpha, \beta, \rho)$ equals

$$\sum_{d=1}^{i-1}\Omega_{d,k}\left(\frac{p_{rd}^k}{p_{sr}}\right)^d + \sum_{d=i}^{M}\Omega_{d,k}\left(\frac{p_{rd}}{p_{sr}}\right)^{d-i}\left(\frac{p_{rd}^k}{p_{sr}}\right)^i. \tag{77}$$

Regarding $\frac{p_{rd}}{p_{sr}} = \frac{\alpha\delta_r}{\delta_s\beta}$ as a finite constant greater than one, then (77) is dominated by the event of $d = 1$ when $\frac{p_{rd}^k}{p_{sr}} \ll 1$ at high SNR, which results in $\Upsilon_k(\alpha, \beta, \rho) \doteq Mp_{sr}^{-1}p_{rd}^k$. As for $n > k' \ge k \ge 1$, however, we have $p_{rd}^k \ge p_{sr}$, namely, $1 > (\frac{\delta_r}{\beta\rho})^k \ge \frac{\delta_s}{\alpha\rho}$. Then, $\Upsilon_k(\alpha, \beta, \rho)$ becomes dominated by the second term on the RHS of (77), yielding

$$\sum_{k=1}^{n} \Lambda_{\alpha,\rho}^{n-k+1} \Upsilon_k(\alpha,\beta,\rho) = \sum_{k=1}^{k'} \sum_{d=i}^{M} \Omega_{d,k} \frac{p_{rd}^d}{p_{sr}^d} \left[p_{sd} p_{sr}^M\right]^n \left[\frac{p_{rd}^i}{p_{sd} p_{sr}^M}\right]^{k-1} + \sum_{k=k'+1}^{n} M \frac{p_{rd}}{p_{sr}} p_{sd}^n p_{sr}^{Mn} \left[\frac{p_{rd}}{p_{sd} p_{sr}^M}\right]^{k-1} \doteq M p_{sd} p_{sr}^{M-1} p_{rd}^n \tag{78}$$

whose order is of ρ^{M+n} due to the dominant event of $k = n$.

In case of $p_{rd}^n \geq p_{sr}$, i.e., $k' \geq n$, then (78) becomes

$$\sum_{k=1}^{n} \sum_{d=i}^{M} \Omega_{d,k} \frac{p_{rd}^d}{p_{sr}^d} \left(p_{sd} p_{sr}^M\right)^n \left(\frac{p_{rd}^i}{p_{sd} p_{sr}^M}\right)^{k-1} \doteq p_{sd} p_{sr}^M p_{rd}^{i(n-1)} \sum_{d=i}^{M} \Omega_{d,n} \frac{p_{rd}^d}{p_{sr}^d}. \tag{79}$$

This is clearly of the order of $\rho^{M+1+i(n-1)}$, and by (75), we obtain $\mathbb{P}_{A,i}(n) \doteq p_{sd} p_{sr}^M p_{rd}^{i(n-1)} \sum_{d=i}^{M} \Omega_{d,n} \frac{p_{rd}^d}{p_{sr}^d}$ since it is impractical to have $p_{sd} \Lambda_{\alpha,\rho}^n > p_{sd} p_{sr}^M p_{rd}^{i(n-1)} \sum_{d=i}^{M} \Omega_{d,n} \frac{p_{rd}^d}{p_{sr}^d}$ in (75), which in general would lead to

$$p_{sd}^n p_{sr}^{(M-i)(n-1)} > \sum_{d=i}^{M} \Omega_{d,n} \left(\frac{p_{rd}}{p_{sr}}\right)^{d+i(n-1)} > 1. \tag{80}$$

It is noted that k' changes with ρ. In different ranges of the SNR, the logarithm of $\sum_{k=1}^{n} \Lambda_{\alpha,\rho}^{n-k+1} \Upsilon_k(\alpha,\beta,\rho)$ may, thus, be dominated by the logarithm of (78) or (79), depending on their relative magnitudes. In other words, it either has a slope of $-[M+1+i(n-1)]$ w.r.t. $\log\rho$ when $p_{rd} \geq [\sum_{d=i}^{M} \frac{\Omega_{d,n}}{M} (\frac{\alpha}{\delta_s} \frac{\delta_r}{\beta})^{d-1}]^{\frac{-1}{(n-1)(i-1)}} \triangleq p_{A_1}$ and $p_{rd} \geq p_{sr}^{\frac{1}{n}}$ or, otherwise, a slope of $-(M+N)$, given $p_{rd} \geq p_{sr}$, i.e., $\frac{\alpha}{\delta_s} \geq \frac{\beta}{\delta_r}$.

3) For $i = 1$, on the other hand, it follows from (75) that

$$\sum_{k=1}^{n} \Lambda_{\alpha,\rho}^{n-k+1} \Upsilon_k(\alpha,\beta,\rho) \doteq \sum_{d=1}^{M} C_d^M p_{sd} p_{sr}^{M-d} p_{rd}^{d+n-1}. \tag{81}$$

In this case, $\mathbb{P}_{A,i}(n)$ will be dominated by the first term on the RHS of (75) only if

$$p_{sd}^{n+1} p_{sr}^{Mn} > \sum_{d=1}^{M} C_d^M p_{sd} p_{sr}^{M-d} p_{rd}^{d+n-1}. \tag{82}$$

Namely, $p_{sr} > p_{A_3} \triangleq [\frac{1}{\Delta_{\alpha,\beta}} (\frac{\delta_s}{\delta_r}\beta)^n]^{\frac{-1}{M(n-1)+1}} \geq p_{A_2}$ since $\Delta_{\alpha,\beta} \triangleq \sum_{d=1}^{M} C_d^M (\frac{\alpha}{\delta_s} \frac{\delta_r}{\beta})^{d-1} \geq M$.

Appendix E
Proof of Proposition 5

Since $\{\underline{d}_\ell + i_\ell(M - \underline{d}_\ell)\} \geq M$, with the equality holding either when $i_\ell \triangleq \min\{i, \underline{d}_{\ell-1}\} = 1$ or $\underline{d}_\ell = M$. Given $i > 1$ and $d_0 \geq 1$, for $k \geq 2$, by inspection, we have the following:

1) Define $\mathcal{U}_0 \triangleq \{d_0 | d_0 \in [1, M]\}$.

2) ARQ2:
   To satisfy $\underline{d}_1 + i_1(M - \underline{d}_1) = M$:
   a) For ($i_1 = 1$ and $\underline{d}_1 < M$), then $\underline{d}_0 = 1$, we have $\mathcal{U}_1^a \triangleq \{(d_1, d_0) | d_0 = 1, d_1 \in [0, M-2]\}$;
   b) For $\underline{d}_1 = M$. Since $\underline{d}_0 \geq 1$, we have $\mathcal{U}_1^b \triangleq \{(d_1, d_0) | d_1 = M - \underline{d}_0, d_0 \in \mathcal{U}_0\}$;
   c) Thus, $\mathcal{U}_1 \triangleq \mathcal{U}_1^a \cup \mathcal{U}_1^b$ satisfies $\underline{d}_1 + i_1(M - \underline{d}_1) = M$.

3) ARQ3:
   a) For ($i_2 = 1$ and $\underline{d}_2 < M$), then $\underline{d}_1 = 1$. Since $d_0 \geq 1$, $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M$, $\ell \in [1, 2]$, is satisfied when $\mathcal{U}_2^a \triangleq \{(d_2, d_1, d_0) | d_0 = 1, d_1 = 0, d_2 \in [0, M-2]\}$.
   b) For $\underline{d}_2 = M$, to also satisfy $\underline{d}_1 + i_1(M - \underline{d}_1) = M$, we have $(d_1, d_0) \in \mathcal{U}_1$. Thus, $\mathcal{U}_2^b \triangleq \{(d_2, d_1, d_0) | d_2 = M - \underline{d}_1, (d_1, d_0) \in \mathcal{U}_1\}$.
   c) $\mathcal{U}_2 \triangleq \mathcal{U}_2^a \cup \mathcal{U}_2^b$ satisfies $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M$, $\ell \in [1, 2]$.

4) For ARQk-1:
   Assume $\mathcal{U}_{k-2} \triangleq \mathcal{U}_{k-2}^a \cup \mathcal{U}_{k-2}^b$ satisfies $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M \; \forall \ell \in [1, k-2]$, with $\mathcal{U}_{k-2}^a \triangleq \{(d_{k-2}, \ldots, d_0) | d_0 = 1, d_1 = \cdots = d_{k-3} = 0, d_{k-2} \in [0, M-2]\}$ and $\mathcal{U}_{k-2}^b \triangleq \{(d_{k-2}, \ldots, d_0) | d_{k-2} = M - \underline{d}_{k-3}, (d_{k-3}, \ldots, d_0) \in \mathcal{U}_{k-3}\}$.

5) For ARQk:
   a) For ($i_{k-1} = 1$ and $\underline{d}_{k-1} < M$), then $\underline{d}_{k-2} = 1$. Since $d_0 \geq 1$, $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M \forall \ell \in [1, k-1]$ is satisfied when $\mathcal{U}_{k-1}^a \triangleq \{(d_{k-1}, \ldots, d_0) | d_0 = 1, d_1 = \cdots = d_{k-2} = 0, d_{k-1} \in [0, M-2]\}$.
   b) For $\underline{d}_{k-1} = M$, to also satisfy $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M \forall \ell \in [1, k-2]$, we have $(d_{k-2}, \ldots, d_0) \in \mathcal{U}_{k-2}$. Thus, $\mathcal{U}_{k-1}^b \triangleq \{(d_{k-1}, \ldots, d_0) | d_{k-1} = M - \underline{d}_{k-2}, (d_{k-2}, \ldots, d_0) \in \mathcal{U}_{k-2}\}$.
   c) Having $\mathcal{U}_{k-1} \triangleq \mathcal{U}_{k-1}^a \cup \mathcal{U}_{k-1}^b$ satisfies $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M \forall \ell \in [1, k-1]$.

By induction, the proof is completed.

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