Effective Protocols and Channel Quality Control Mechanisms for Cooperative ARQ With Opportunistic AF Relaying

Chun-Kai Tseng and Sau-Hsuan Wu

Abstract—**Incorporating relaying techniques into automatic repeat reQuest (ARQ) in general will provide diversity and throughput enhancements. However, when opportunistic amplify-and-forward (AF) relaying is applied to cooperative ARQ, the system design becomes much more involved. First, our capacity outage analysis shows that the temporal diversities of ARQs with a single AF relay cannot be exploited unless the channel quality to the relay exceeds a threshold. This notion of selective AF relaying is extended to systems with multiple relays in an attempt to jointly explore the temporal and spatial diversities with ARQ. Two types of selective and opportunistic AF relaying schemes are then developed for such kinds of relay-assisted ARQ. And our analysis further shows that the temporal and spatial diversities cannot be fully exploited without the use of overhearing among relays and a proper link quality control mechanism to prescreen the overheard signals. This quality control mechanism is implemented with a set of thresholds designed for each hop of the relaying path. Feasible threshold setting methods are also developed for the proposed ARQ protocols to achieve their potential diversities. In contrast to our designs, the ARQ scheme with the typical opportunistic AF relaying method suffers from severe diversity losses. Simulations also show that the proposed ARQ schemes are more effective in throughput enhancement, and can provide cell-edge users almost three times the throughput gain in comparison with ARQ with no relay-assisted forwarding.**

Index Terms—**Cooperative ARQ, link quality control, opportunistic AF relaying, selective AF relaying, and SOAF.**

I. Introduction

CELL-EDGE signal quality, which generally suffers from low reception power and severe co-channel interference, has been a key issue for wireless communication system designs. This signal quality control problem will become more complicated if the density of wireless networks becomes higher. Cooperative relaying, enabling relay stations to support data transmissions, provides an alternative and cost-effective approach to enhance signal coverage and transmission reliability.

Since the works of [1]–[3], a host of relaying protocols have been studied and presented, either in academia (e.g., [4]–[11] and the reference therein), or in international standards, such as 3GPP LTE-Advanced (LTE-A) [12] and IEEE802.16j [13]. Among the works, amplified-and-forward (AF) and decoded-and-forward (DF) are the two mostly studied relaying methods. In contrast to DF relaying, AF relaying which only needs to forward the amplified received signals has a lower cost, smaller signal processing delays, and a simpler system requirement in deployment [14]. In spite of these operational advantages, AF relaying inevitably causes noise enhancement in relayed signals. To control the effects of noise propagations, selective AF (SAF) relaying methods are proposed in [15] to enhance the power efficiency of AF relaying, and in [10] to improve the performance of multi-hop relaying. Basically, a relay in the SAF scheme is activated only if the source-to-relay (S-R) channel quality is greater than a predetermined threshold.

On the other hand, to avoid the complexity of using distributed space-time coding [2] or beamforming, an opportunistic relaying (OR) strategy is proposed in [3] to exploit the spatial diversity offered by multiple AF relays. This opportunistic AF (OAF) relaying method uses the relay with the highest instantaneous signal-to-noise ratio (SNR) from the source to the destination (S-D) to forward the signal, and can exploit the full spatial diversity offered by multiple relays in a typical two-hop relaying manner.

Nevertheless, the diversity is gained at the expense of an extra relaying phase for every packet transmission, which limits the spectral efficiency of this diversity technique. To circumvent this problem, one may use non-orthogonal or full duplex relaying that allows the source node to continue sending new packets while the relay nodes forward the old ones, e.g., [5]–[8]. This approach typically requires a fairly high complexity in data detection. In contrast, a more straightforward method is to incorporate cooperative relaying in automatic repeat request (ARQ). This cross-layer method allows relays to forward signals only if the direct-link transmissions fail. In other words, relaying is used for retransmissions only. This ARQ scheme in principle provides a higher retransmission reliability and thus reduces the average packet delivery delay, leading to a throughput enhancement [16].

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In view of the potential of relaying for ARQ, several AF relaying based ARQ protocols have been presented [16]–[20]. In [17], an adaptive relaying scheme is proposed where AF relays sequentially forward their received signals in different time slots until the destination successfully decodes the packet. To explore a higher spatial diversity, an OAF relaying based ARQ is proposed in [16], where each ARQ round is done by a selected relay that can offer the highest S-D end-to-end SNR. Nevertheless, our analysis will soon show that the spatial and temporal diversities available from ARQ are in fact not utilized by the typical OAF ARQ scheme after the first round of retransmission due to noise propagation and the lack of quality control on the relayed signals. A similar problem also occurs in the path selection method of [11] for multi-hop AF relaying. For other AF based ARQ schemes, e.g., [18]–[20], then two-hop transmissions are typically used in every ARQ round in order to avoid the complicated quality control problem on the different hops of relaying paths in ARQ, which, however, leads to capacity losses due to the source rebroadcasting and the relaying steps involved in every ARQ.

The above results show that the typical noise enhancement problem in AF relaying appears to be a challenging issue in the designs of AF based ARQ protocols that aim to use OR to exploit the spatial and temporal diversities altogether. As to be shown in this work, the design problem is particularly involved due to the noise propagation effects in the relay reselection and retransmission processes of ARQ. On one hand, the temporal diversity offered by retransmissions is limited by the noise coupled in the amplified and relayed signals. On the other hand, the spatial diversity is dominated by the worst S-R channel quality in retransmissions even if OR is employed to reselect the best relay in every ARQ.

To resolve these problems, one not only needs to overcome the noise propagation effects in AF relaying, but also needs to prevent the spatial diversity from being limited by the worst S-R channel quality. The key lies in a delicate screening process to avoid unqualified relays from being selected in each single ARQ, in the meantime to protect relays with reasonably good channel qualities from being screened out, and in an overhearing and rejuvenation process to reactivate the unqualified relays for subsequent ARQs. This implies that an effective screening mechanism, like SAF relaying, comes hand in hand with an OR-based ARQ scheme that has the potential to provide the full spatial and temporal diversities. To unravel these intertwined issues in the screening, reselection and reactivation processes, we adopt a divide-and-conquer approach to solve each individual problem step by step.

In essence, to resolve the diversity loss in the typical OAF ARQ [16] while avoiding the two-hop transmission involved in every ARQ round [18]–[20], we develop a new type of ARQ based on a selective and opportunistic AF (SOAF) relaying method, and design link quality control mechanisms for it. Different from the typical OAF relaying [3], the SOAF relaying only requires channel qualities at the receiving ends. The main results and contributions of this research study are highlighted as follows:

1) At first, we provide an outage analysis to show that the thresholding method of SAF relaying also plays a key role for ARQs that use AF relaying to exploit the temporal diversity. If the threshold is not properly set to screen out unqualified relays in advance, then an ARQ will fail to make use of the temporal diversity from channel variations. This implies that the typical OAF relaying method [3] is not able to utilize the spatial diversity from OR after the first round of ARQ, either.

2) To combat this noise propagation effect, we first provide a thresholding method that allows AF relaying to continue exploiting the temporal diversity through ARQs. Different from the typical SAF or the OAF relaying, the proposed method employs a selective and opportunistic AF (SOAF) relaying mechanism to control channel qualities on both the source and the forward links of the AF relaying nodes, respectively, yet only requires channel qualities on the receiving ends, which much simplifies the implementation of the proposed scheme.

3) Extending this result, we then devise an advanced version of the SOAF relaying method for ARQ to exploit the full temporal and spatial diversities offered by multiple relays. The basic idea originates from overhearing, which allows unqualified relays to overhear the signals forwarded from qualified relays, and a stricter thresholding mechanism to screen relays in every step of the qualification and selection process of the every hop of AF relaying. The quality control on the S-R or the relay-to-relay (R-R) links of different hops is served by a set of thresholds designed for each hop, and is done by the relays themselves. And the main challenge to the thresholds design is to exploit the diversity and the SNR gains from relays' overhearing at the same time. Thresholds too low will result in diversity losses, while thresholds too high will lead to SNR gain losses as well. Two feasible thresholds setting methods are then proposed and discussed in this work.

4) Although, quality control on the relay links is typically not a concern in DF based ARQ schemes [21]–[25]. It turns out to be a fundamental problem in the AF based schemes. Identifying a key property from our analysis, we provide in this paper a new look and method for quality control along each hop of the multi-AF relaying systems. The proposed quality control method is shown effective to exploit the full spatial diversity in every ARQ round, yet is able to provide a performance comparable to that of the DF based counterparts. Simulation studies also show that our proposed ARQ schemes can more effectively enhance the cell-edge throughput than the OAF ARQ, and can provide almost three times the cell-edge throughput higher than ARQ with no relay forwarding.

The paper is organized as follows. Section II introduces the basic system model. In Section III, we analyze the outage probability of an ARQ scheme that uses single-relay SAF relaying. Extending this result, we provide in Section IV the outage analysis for ARQ schemes that use OR, from which two types of SOAF ARQ protocols are developed. Section V studies threshold setting methods for the proposed ARQ schemes, followed by their throughput simulation results in Section VI.

Notations: $\mathbb{R}$ and $\mathbb{C}$ stand for the real and the complex fields respectively, $\mathbb{R}^+$ represents the positive real field, and $\mathbb{N}$ denotes

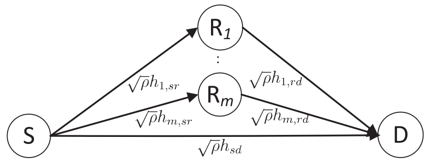


Fig. 1. An illustration of the considered multi-AF-relay system.

the set of natural numbers. $\mathbb{E}_{a>0}[f(a)]$ denotes the expectation of $f(a)$ over the region of the random variable $a > 0$. The term $a := b$ means "b" is assigned to "a". In general, $|a|$ stands for the absolute value of the variable "a", but if "a" is a set, $|a|$ represents its cardinality. We denote the index set of $[i, i+1, \ldots, j]$ by $\mathcal{I}_i^j$ where i and j are integers. The diversity order d of a function $f(\rho)$ is defined as $d \triangleq -\lim_{\rho\to\infty} \log f(\rho)/\log \rho$, and $f(\rho) \overset{d}{=} g(\rho)$ means they have the same diversity order. Finally, $f(\rho) \doteq g(\rho)$ represents $\lim_{\rho\to\infty} \rho^d f(\rho) = \lim_{\rho\to\infty} \rho^d g(\rho)$ with the exponent d equal to the diversity order of $f(\rho)$ and $g(\rho)$. Following this definition, we may define $\dot{\leq}$ and $\dot{\geq}$ accordingly. Other notations for analysis will be defined in the sequel when needed.

II. Basic System Model

We consider a relay-assisted wireless network, as illustrated in Fig. 1, which consists of one source, one destination and m AF relays. The relays are employed here to help retransmit signals. The source, destination and relays are assumed equipped with a single antenna. We denote the channel coefficients from the source to the destination, the source to relay j, and the relay to the destination by h_{sd}, $h_{j,sr}$ and $h_{j,rd}, \forall j \in \mathcal{I}_1^m$. Throughout the paper, the channels between any transmit and receive pairs are considered flat and block Rayleigh faded, where the corresponding channel coefficients are all zero-mean complex Gaussian random variables whose realizations remain unchanged within a packet duration of L symbols, and change independently for every packet transmission or retransmission.

This block-fading assumption is reasonable under the situation [26]: *Within a network that serves multiple users over time-variant channels, such as time division multiple access (TDMA) systems, the length of each packet to be transmitted or received by one user is less than the channel coherence interval, and every packet belonging to a certain user would not be processed in continuous phases or resource blocks since each user should be fairly served*. Though stated for a TDMA system more than a decade ago, the same operation condition applies to the current 4G system with orthogonal frequency division multiple access (OFDMA). Despite the fact that an exact analysis for a large network such as LTE-A is beyond the scope of this paper, the single-user analysis conducted herein under the block-fading assumption can provide valuable insights into the multi-user systems described above.

In addition, for the simplicity of expression, the average transmit SNR is assumed, without loss of generality, to be the same at the source and the relays, and is denoted by $\rho \triangleq \frac{E_s}{N_0}$ where E_s and N_0 stand for the symbol energy and the noise variance. We employ a channel codebook of rate $\mathcal{R}$ (in bits/channel use) with the codeword length equal to the packet length, L. The total number of codewords, $\bar{x}_j \in \mathbb{C}^{L\times 1}$, is thus equal to $\lfloor 2^{\mathcal{R}L} \rfloor$. The codewords are assumed equiprobable, and satisfy an average power constrain of $\mathbb{E}[\|\bar{x}\|^2] = L$.

Further, we define the maximal number of ARQ rounds to be N, and denote the i-th ARQ round by ARQi with ARQ0 standing for the initial packet transmission from the source. The capacity outage probability after n rounds of ARQs with a relaying scheme A is denoted by $\mathcal{P}_{\text{out},n}^{A}$. Finally, to focus on the diversity analysis for cooperative ARQs, the impact of transmission failures of control messages, e.g., acknowledgement (ACK) or negative ACK (NACK), is ignored in this paper, and there is no full channel state information (CSI) available at the source for instantaneous transmission rate adaptation.

A. Relay-Assisted ARQ Model

The ARQ protocols to be investigated or developed basically work as follows. In the beginning of a packet transmission, the source broadcasts its signal to the relays and the destination. If the destination successfully decodes the packet, then it will feed back an ACK signal to the source and the relays, and the source will continue to send the next packet. Otherwise, the destination will issue a NACK signal, which invokes an ARQ procedure for retransmissions. In general, retransmissions will be performed by relays without the source's rebroadcasting once the signals received by at least one of the relays exceed a certain quality level[1]; otherwise, the source will retransmit the signal by itself. The relays' forwarding control also depends on the operational rule of the employed ARQ scheme.

To avoid the need for symbol-level transmission synchronization among multiple points, such as using distributed space-time coding, we consider an opportunistic relay selection method where at most one relay is selected for each packet retransmission. Compared to the typical OAF relay selection method [3], [16], the method adopted herein only requires the relay-to-destination (R-D) instantaneous CSI, which makes it easier to design our proposed ARQ schemes in practice. For instance, to implement this method in a cellular network, such as LTE-A [12], the participating relays will be requested by the base station to send pilot signals in dedicated relay subframes to let the destination identify the relay that has the highest $\rho|h_{j,rd}|$ and report the relay's identification number to the base station for retransmission scheduling.

III. ARQ with Selective AF Relaying

We start our analysis with a single relay case, i.e., $m = 1$. This case is aimed to show how the selective relaying scheme influences the exploitation of the temporal diversity offered by retransmissions. In particular, we will show from the viewpoint of outage probability that the diversity order of this SAF based ARQ scheme is limited to two if the threshold on the S-R channel quality is not properly set, regardless of the number of ARQs. On the contrary, if the threshold is set high enough, then the

[1]The threshold level(s) studied later for our proposed schemes can be easily calculated and acquired by the relays before an initial packet transmission.

temporal diversity offered by retransmissions can be utilized by ARQs with SAF relaying. Nevertheless, a very high threshold may result in a loss of SNR gain. The analysis and notion established here will be later extended in Section IV to design ARQ schemes ($m > 1$) that use both SAF and OR to exploit the temporal and spatial diversities altogether.

A. The Outage Probability of ARQs with SAF Relaying

Different from the typical AF relaying function, at the beginning of a source packet transmission, the relay of the SAF scheme first senses its S-R link quality, $\rho|h_{sr}|^2$, and compares it against a predetermined threshold, Δ. If $\rho|h_{sr}|^2 > \Delta$, the relay further receives and records the noise-corrupted source signal in its buffer, and then labels itself as a qualified node that is ready for retransmission. Otherwise, the relay remains idle. In case of an initial reception failure at the destination, if the relay is qualified, it then amplifies and forwards its recorded signal to the destination for ARQ. And, the relay will continue to use the same recorded signal for the following ARQs of the same packet if needed. Otherwise, the source retransmits the packet by itself until the relay becomes qualified, or when no ARQ is further needed, i.e., either the packet is delivered successfully or when the maximal number of ARQs is reached.

Following this ARQ protocol, if the source retransmits for ARQl, during which the relay is also assumed to first become qualified, then we can model the received signals at the destination and the relay in this ARQ round, respectively, as

$$\bar{y}_{sd}^{(l)} = \sqrt{\rho}h_{sd}^{(l)}\bar{x} + \bar{n}_d^{(l)} \text{ and } \bar{y}_{sr}^{(l)} = \sqrt{\rho}h_{sr}^{(l)}\bar{x} + \bar{n}_r^{(l)} \quad (1)$$

where the subscript "(l)" is used to indicate the retransmission block of ARQl, and the entries of the noise vectors $\bar{n}_d$ and $\bar{n}_r$ are independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and unit variance.

For the following ARQs, i.e., ARQi with $i > l$, the retransmissions are performed by the relay. Thus, the corresponding received signals at the destination can be expressed as

$$\bar{y}_{rd}^{(i)} = \frac{\sqrt{\rho}h_{rd}^{(i)}}{\sqrt{\rho|h_{sr}^{(l)}|^2+1}}\bar{y}_{sr}^{(l)} + \bar{n}_d^{(i)}, \text{ for } i > l \quad (2)$$

where the denominator $(\rho|h_{sr}^{(l)}|^2 + 1)$ is the power normalization factor [1] used by the relay on its received signal $\bar{y}_{sr}^{(l)}$.

We denote the instantaneous received SNR at the destination for ARQi by $\Gamma_d^{(i)}$. Following the models (1) and (2) where the source retransmits for ARQ0$\sim$ARQl, we have $\Gamma_d^{(i)} = \rho|h_{sd}^{(i)}|$ for $i = 0, \ldots, l$, and $\Gamma_d^{(i)}$ given by [1], [27]

$$\Gamma_d^{(i)} = \frac{\rho^2|h_{sr}^{(l)}|^2|h_{rd}^{(i)}|^2}{\rho|h_{sr}^{(l)}|^2 + \rho|h_{rd}^{(i)}|^2 + 1}, \text{ for } i > l. \quad (3)$$

We note here that the same S-R link quality $\rho|h_{sr}^{(l)}|^2$ is coupled in the relayed signals of ARQs.

Based on the aforementioned system model and ARQ-SAF protocol, we show in what follows that the threshold Δ for SAF relaying plays a crucial role for an ARQ to achieve its full diversity. In contrast, the ARQ scheme with the typical AF relaying ($\Delta = 0$) is not able to make use of the temporal diversity from retransmissions. The analysis is mainly based on the capacity outage probability of the form

$$\Pr\left\{\log_2\left(1+\Gamma_d^{(i)}\right) < \mathcal{R}\right\} = \Pr\left\{\Gamma_d^{(i)} < \delta\right\} \quad (4)$$

with $\delta \triangleq 2^{\mathcal{R}} - 1$. In addition, for convenience of exposition, we define a few notations to be used frequently in the analysis. First, we define the variances of the channels h_{sd}, h_{sr}, and h_{rd} as β_0, β_1 and β_2. Radio propagation losses are considered in the channel variances. We then define some random variables as $w \triangleq \rho|h_{sd}|^2 \sim \text{Exp}(\rho\beta_0)$, $a \triangleq \rho|h_{sr}|^2 \sim \text{Exp}(\rho\beta_1)$, and $b \triangleq \rho|h_{rd}|^2 \sim \text{Exp}(\rho\beta_2)$ where we ignore the transmission block indices of the channel coefficients for conciseness and will specify them if necessary, and $x \sim \text{Exp}(y)$ means x is exponentially distributed with a mean equal to y.

Let $F(\Delta, \ell)$ stand for the capacity outage probability of ℓ consecutive retransmissions by the relay. Given the threshold Δ, the outage probability after n rounds of ARQs with the SAF relaying scheme can be expressed as

$$\begin{aligned}\mathcal{P}_{\text{out},n}^{\text{SAF}} &= \Pr\{w < \delta\} \\ &\times \sum_{\ell=0}^{n}\Big[(\Pr\{a \le \Delta\}\Pr\{w < \delta\})^{n-\ell}F(\Delta, \ell)\Big] \quad (5)\end{aligned}$$

where the first term $\Pr\{w < \delta\}$ results from ARQ0, and the term $(\Pr\{a \le \Delta\}\Pr\{w < \delta\})^{n-\ell}$ corresponds to the cases of $n - \ell$ consecutive retransmissions by the source. As for the following ℓ ARQ rounds done by the relay, based on the form of (3), $F(\Delta, \ell)$ for $\ell > 0$ is defined as the joint probability of

$$F(\Delta, \ell) = \Pr\left\{a > \Delta, \tfrac{ab_1}{a+b_1+1} < \delta, \ldots, \tfrac{ab_\ell}{a+b_\ell+1} < \delta\right\} \quad (6)$$

with $F(\Delta, 0) \triangleq 1$. Since the R-D channel is assumed to fade independently in each ARQ round, the subscript of b_ℓ is used to distinguish the channel quality in each ARQ round.

Obviously, the retransmission events in $F(\Delta, \ell)$ are correlated since the received S-R channel quality "a" in them are the same even if b_l are statistically independent. With some mathematical manipulations, it can be further shown that the form of $F(\Delta, \ell)$ depends on the ratio of Δ to δ, and can be expressed as a formula summarized in the next lemma.

Lemma 1: Given Δ, $\mathcal{R}$ and, hence, δ, we have

$$F(\Delta, \ell) = e^{-\frac{\Delta}{\rho\beta_1}} + \begin{cases}\sum_{i=1}^{\ell}\mathcal{C}_i^\ell(-1)^i e^{-(\frac{1}{\rho\beta_1}+\frac{i}{\rho\beta_2})\delta}\Gamma(1, 0; \frac{i(\delta^2+\delta)}{\rho^2\beta_1\beta_2}), & \Delta < \delta \\ \sum_{i=1}^{\ell}\mathcal{C}_i^\ell(-1)^i e^{-(\frac{1}{\rho\beta_1}+\frac{i}{\rho\beta_2})\delta}\Gamma(1, \frac{\Delta-\delta}{\rho\beta_1}; \frac{i(\delta^2+\delta)}{\rho^2\beta_1\beta_2}), & \Delta \ge \delta\end{cases} \quad (7)$$

where $\Gamma(\alpha, x; b) \triangleq \int_x^\infty t^{\alpha-1}e^{-t-\frac{b}{t}}dt$ is the generalized incomplete gamma function [28], and $\mathcal{C}_i^\ell \triangleq \frac{\ell!}{i!(\ell-i)!}$.

Proof: See Appendix A. ∎

Substituting (7) into (5) gives the exact expression for $\mathcal{P}_{\text{out},n}^{\text{SAF}}$. Let $\Delta := \lambda \times \delta$ with $\lambda \in \mathbb{R}^+$. Given $\mathcal{R}$, i.e., δ, the relations between λ and the SNR ρ for $\mathcal{P}_{\text{out},n}^{\text{SAF}}$ to achieve a target P_t are illustrated in Fig. 2. As can be seen in the figure, the required SNRs with different P_t's all reduce dramatically around $\lambda = 1$. This in fact results from the diversity variations with respect

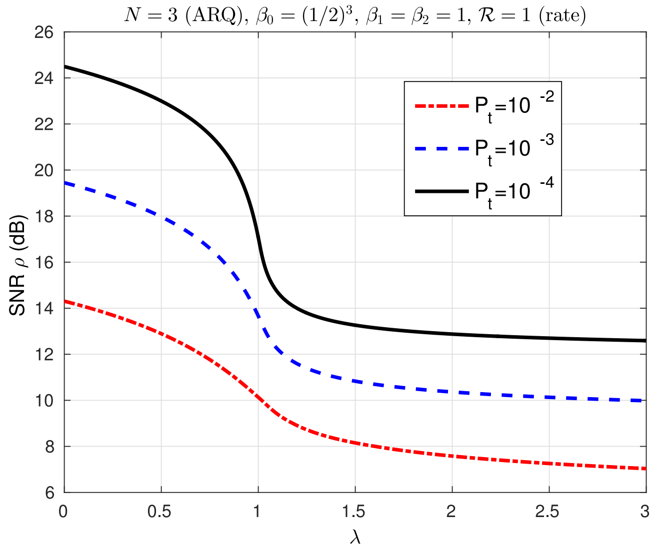


Fig. 2. Given δ, the relations between λ and the transmit SNR ρ for $\mathcal{P}^{\text{SAF}}_{\text{out},3}$ to achieve the target P_t when $\Delta := \lambda\delta$.

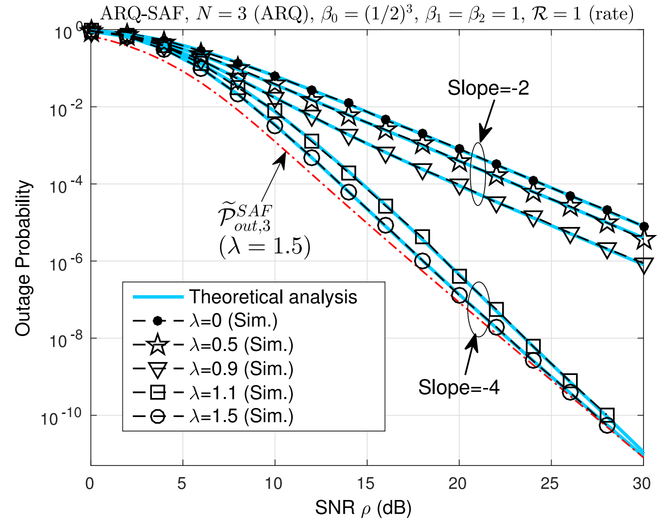


Fig. 3. Outage probabilities after 3 rounds of ARQ-SAFs ($\mathcal{P}^{\text{SAF}}_{\text{out},3}$), with different values of λ when $\Delta := \lambda\delta$.

to (w.r.t.) Δ in $F(\Delta, \ell)$ of $\mathcal{P}^{\text{SAF}}_{\text{out},n}$, which is analyzed in the next lemma. Before that, we introduce a useful lower bound for $F(\Delta, \ell)$, denoted by $\widetilde{F}(\Delta, \ell)$, which is defined as

$$\widetilde{F}(\Delta, \ell) \triangleq \Pr\{a > \Delta\} \Pr\{b_1 < \delta, \ldots, b_\ell < \delta\}, \ \ell \geq 1 \quad (8)$$

with $\widetilde{F}(\Delta, 0) \triangleq 1$. Comparing (8) to (6), we have $F(\Delta, \ell) \geq \widetilde{F}(\Delta, \ell)$ for any Δ since $\frac{ab}{a+b+1} \leq \min[a, b] \leq b$ holds $\forall a, b \geq 0$. Namely, we obtain the lower bound of $F(\Delta, \ell)$ by ignoring the effect of noise enhancement on the relayed signals.

Lemma 2: Given the rate $\mathcal{R}$, namely δ, let $\Delta := \lambda \times \delta$ with $\lambda \in \mathbb{R}^+$. For $\ell \in \mathbb{N}$, we have $F(\Delta, \ell) \doteq \widetilde{F}(\Delta, \ell) \doteq (\frac{\delta}{\rho\beta_2})^\ell$ if $\lambda > 1$; whereas, if $\lambda < 1$, $F(\Delta, \ell)$ is of the order of ρ^{-1}, and follows $F(\Delta, \ell) \doteq \frac{\delta-\Delta}{\rho\beta_1}$ for $\ell \geq 2$.

Proof: See Appendix B. ■

Intuitively, as $\lambda > 1$, the thresholding mechanism prevents low-quality signals from being relayed, as such the ARQ events become virtually uncorrelated with the S-R link quality "a" at high SNR, leading to $F(\Delta, \ell) \doteq \widetilde{F}(\Delta, \ell)$. In addition, $\widetilde{F}(\Delta, \ell)$ is indeed a useful approximation for $F(\Delta, \ell)$ since no numerical integration is needed for the evaluation of $\Gamma(\alpha, x; b)$ in (7). In view of the simplicity, we then define a lower bound $\widetilde{\mathcal{P}}^{\text{SAF}}_{\text{out},n}$ for $\mathcal{P}^{\text{SAF}}_{\text{out},n}$ by replacing $F(\Delta, \ell)$ in (5) with $\widetilde{F}(\Delta, \ell)$.

Since $(\Pr\{a \leq \Delta\} \Pr\{w < \delta\})$ in $\mathcal{P}^{\text{SAF}}_{\text{out},n}$ is equal to $(1 - e^{-\frac{\Delta}{\rho\beta_1}})(1 - e^{-\frac{\delta}{\rho\beta_0}}) \doteq \frac{\Delta}{\rho\beta_1}\frac{\delta}{\rho\beta_0}$ whose diversity order is two, by (5) and Lemma 2, we can see that $\mathcal{P}^{\text{SAF}}_{\text{out},n}$ will be dominated at high SNR by the case of $\ell = n$ in (5) due to its lowest diversity order; namely $\mathcal{P}^{\text{SAF}}_{\text{out},n} \doteq \Pr\{w < \delta\} F(\Delta, n)$. The relationship between $\mathcal{P}^{\text{SAF}}_{\text{out},n}$ and Δ can thus be characterized as follows:

Proposition 1: Let $\Delta := \lambda\delta$ with $\lambda \in \mathbb{R}^+$. If $\Delta > \delta$, we have $\mathcal{P}^{\text{SAF}}_{\text{out},n} \doteq \widetilde{\mathcal{P}}^{\text{SAF}}_{\text{out},n} \doteq \frac{\delta}{\beta_0}(\frac{\delta}{\beta_2})^n \rho^{-(n+1)}$; whereas, if $\Delta < \delta$, we arrive at $\mathcal{P}^{\text{SAF}}_{\text{out},n} \doteq \frac{\delta}{\beta_0}\frac{\delta-\Delta}{\beta_1}\rho^{-2}$, for $n \geq 2$.

The proposition shows that if a basic channel quality is met at the relay before using the AF relaying, the temporal diversity of ARQs can be greatly improved from the viewpoint of outage probability. Simulation results for $\mathcal{P}^{\text{SAF}}_{\text{out},3}$ with different Δ's are shown in Fig. 3 to verify our theoretical analysis. For $\lambda > 1$, $\mathcal{P}^{\text{SAF}}_{\text{out},3}$ becomes closer to $\widetilde{\mathcal{P}}^{\text{SAF}}_{\text{out},3}$ when the SNR increases.

On the other hand, Proposition 1 also shows that the diversity order of ARQs with direct AF relaying (ARQ-AF) is equal to two since it is simply a special case of ARQ-SAF with $\Delta := 0$. Based on (6), the corresponding capacity outage probability for ARQ-AF is given by $\mathcal{P}^{\text{AF}}_{\text{out},n} = \Pr\{w < \delta\} F(0, n)$.

IV. Cooperative ARQ with Selective and Opportunistic AF Relaying

The analysis in the previous section points out the importance and the role of S-R link quality control in our quest to improve the system reliability with AF retransmissions. In addition to utilizing the temporal diversity with ARQs, one may also exploit the spatial diversity with multiple relays and the idea of OAF in [3]. Incorporating the spatial diversity scheme of OAF into the SAF ARQ framework may allow us to jointly exploit the spatial and temporal diversities in multi-relay systems with the same and simple AF relaying method. The outage analysis on this selective and opportunistic manner of AF relaying leads to two types of ARQ schemes. More importantly, it provides a new look and method on the quality control along each hop of multi-AF-relay systems.

To simplify our theoretical analysis, the variances of $h_{j,sr}$ for different j are assumed the same and equal to β_1 which also denotes the variance of h_{sr} in Section III-A. Similarly, the variance of $h_{j,rd}$ is assumed equal to $\beta_2, \forall j \in \mathcal{I}_1^m$. Thus, we also use the link qualities a and b to denote the random variables $\rho|h_{j,sr}|^2$ and $\rho|h_{j,rd}|^2$, when needed. In addition, we denote the channel coefficient between relay i and relay j by $h_{i,j}$, and assume the variance of $h_{i,j}$ is equal to a number denoted by β_3, $\forall i, j$ and $i \neq j$. Though simplified, the above channel assumptions make the following analysis tractable, and allow us to investigate the quality control mechanism from a theoretical perspective. In particular, we may set β_1, β_2 and β_3 respectively as the worst average S-R, R-D and R-R channel gains of a true system. The resultant system performance can thus be considered as the lowest potential performance of the system. Therefore, the following diversity analysis and channel requirements for relays are still valid in a true system.

A. The Outage Probability of ARQ with the Typical Opportunistic AF Relaying (ARQ-OAF)

We first investigate the outage probability of the ARQ scheme that uses the opportunistic AF relaying method (ARQ-OAF). In ARQ-OAF, to make full use of relaying, only the initial packet transmission (ARQ0) is done by the source, and the following retransmissions are done by selected relays. This scheme chooses a relay $j_*^{(i)}$ in each ARQ round, i, that satisfies

$$j_*^{(i)} = \underset{j\in\{1,\ldots,m\}}{\arg\max} \left\{ \frac{\rho^2 |h_{j,sr}^{(0)}|^2 |h_{j,rd}^{(i)}|^2}{\rho |h_{j,sr}^{(0)}|^2 + \rho |h_{j,rd}^{(i)}|^2 + 1} \right\} \tag{9}$$

to directly amplify and forward the signal. Following this selection rule, we summarize the outage probability after n rounds of the OAF based ARQs in the following proposition.

Proposition 2: Given $\mathcal{R}$ and m, the capacity outage probability after n rounds of ARQs with the typical OAF relaying is given by $\mathcal{P}_{\text{out},n}^{\text{OAF}} = \Pr\{w < \delta\} \times (F(0,n))^m$, and its diversity order is limited to $(m+1)$, $\forall n \in \mathbb{N}$.

Proof: For the clarity of representation, we extend here the notations a and b to $a_{i,j}$ and $b_{i,j}$ for the relay j at ARQi. Based on (9), we can have $\mathcal{P}_{\text{out},n}^{\text{OAF}}$ equal to

$$\begin{aligned} &\Pr\{w<\delta\} \Pr\left\{ \max\left(\frac{a_{0,1} b_{1,1}}{a_{0,1} + b_{1,1} + 1}, \ldots, \frac{a_{0,m} b_{1,m}}{a_{0,m} + b_{1,m} + 1} \right) \right. \\ &\left. < \delta, \ldots, \max\left(\frac{a_{0,1} b_{n,1}}{a_{0,1} + b_{n,1} + 1}, \ldots, \frac{a_{0,m} b_{n,m}}{a_{0,m} + b_{n,m} + 1} \right) < \delta \right\} \\ &= \Pr\{w<\delta\} \Pr\left\{ \frac{a_{0,1} b_{1,1}}{a_{0,1} + b_{1,1} + 1} < \delta, \ldots, \frac{a_{0,1} b_{n,1}}{a_{0,1} + b_{n,1} + 1} < \delta \right\} \\ &\times \cdots \times \Pr\left\{ \frac{a_{0,m} b_{1,m}}{a_{0,m} + b_{1,m} + 1} < \delta, \ldots, \frac{a_{0,m} b_{n,m}}{a_{0,m} + b_{n,m} + 1} < \delta \right\} \\ &= Pr\{w<\delta\} (F(0,n))^m. \end{aligned} \tag{10}$$

As for the diversity order analysis, since $F(0,n)$ is of the order of ρ^{-1} by Lemma 2, we thus have $\mathcal{P}_{\text{out},n}^{\text{OAF}} \stackrel{d}{=} \rho^{-(m+1)}$. ■

In fact, the ARQ-OAF scheme offers the full cooperative diversity only for the first ARQ round. In the subsequent ARQs, even though the selected relays can offer the highest end-to-end instantaneous SNR at the destination, the system still suffers from the loss of temporal diversity as will be verified in Fig. 4. Similar to the ARQ-AF scheme, the loss of the temporal diversity mainly results from the unprescreened S-R link qualities $\rho |h_{j,sr}^{(0)}|^2$, $\forall j$. This motivates us to develop ARQ schemes that on one hand, require relays to prescreen their incoming signal qualities, like the SAF relaying method, and on the other hand, allow the destination to opportunistically choose a relay only from the set of relays that pass the screening. This idea leads to two types of ARQ schemes to be presented below, referred to as the type A and B of ARQ-SOAF. We next start with the most straightforward one.

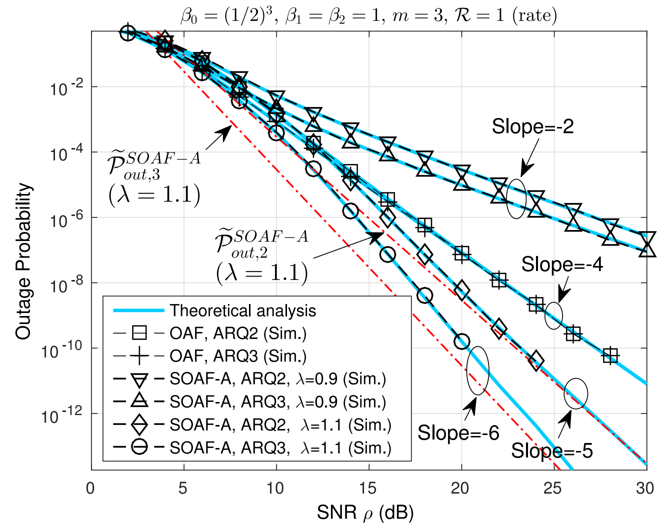


Fig. 4. Outage probabilities for ARQs with OAF and SOAF-A relayings. For SOAF-A with $\Delta := \lambda\delta > \delta$, the diversity orders increase by 1 in each round of the ARQs. Otherwise, they are limited to 2.

B. ARQ with the Type A of SOAF Relaying (SOAF-A)

Extending the idea of the SAF relaying in Section III-A, we define for SOAF relaying a qualified set $\mathcal{Q}$ of the relays that sense their S-R link qualities $\rho|h_{j,sr}|^2 > \Delta$ in the source's broadcasting phase. In each ARQ, the relay in $\mathcal{Q}$ with the highest $\rho|h_{j,rd}|^2$ gets selected to forward its recorded signal. In case of $\mathcal{Q} = \emptyset$, the source will retransmit by itself until $\mathcal{Q} \neq \emptyset$ or when no ARQ is further needed. Compared to the typical OAF scheme of (9), the opportunistic relay selection method here only requires R-D channel gains at the destination, which makes it easier to implement the SOAF scheme in practice.

Specifically, if a $\mathcal{Q}$ with $|\mathcal{Q}| > 0$ first forms at the end of ARQl, then different from (9), the SOAF-A scheme chooses a relay $j_*^{(i)}$ for each ARQi, $i > l$, by the rule following

$$j_*^{(i)} = \underset{j\in\mathcal{Q}}{\arg\max} \left\{ \rho |h_{j,rd}^{(i)}|^2 \right\}. \tag{11}$$

Thus, similar to (3), for each ARQi, $i > l$, we can express in this case the corresponding received SNR at the destination as

$$\Gamma_d^{(i)} = \frac{\rho^2 |h_{j_*^{(i)},sr}^{(l)}|^2 |h_{j_*^{(i)},rd}^{(i)}|^2}{\rho |h_{j_*^{(i)},sr}^{(l)}|^2 + \rho |h_{j_*^{(i)},rd}^{(i)}|^2 + 1}. \tag{12}$$

For (12), we note that the random variable $\rho|h_{j_*^{(i)},sr}^{(l)}|^2$ in fact has the same distribution as $\rho|h_{j,sr}|^2$ for any j and transmission block index (i.e., same as the link quality a), but its realization at ARQl is coupled in the consecutive received SNR at the destination for the following ARQ events. On the other hand, based on (11), we know that $\rho|h_{j_*^{(i)},rd}^{(i)}|^2$ has the same distribution as $\max_{j\in\mathcal{Q}} \rho|h_{j,rd}|^2 \triangleq b^{[q]}$ given $|\mathcal{Q}| = q$. In other words, different cardinalities of the $\mathcal{Q}$ at ARQl will lead to different distributions of the random variable $\Gamma_d^{(i)}$ in (12).

With those observations, we can start to calculate the performance of SOAF-A ARQ. We first let $G_1(\Delta, \ell)$ be the capacity outage probability of ℓ consecutive retransmissions by relays chosen according to the rule of the SOAF-A protocol. The outage probability after n rounds of ARQs with this SOAF-A relaying method can be expressed in a form as below:

Proposition 3:

$$\mathcal{P}_{\text{out},n}^{\text{SOAF-A}} = \Pr\{w < \delta\} \times \sum_{\ell=0}^{n} \Big[(\Pr\{a \leq \Delta\}^m \Pr\{w < \delta\})^{n-\ell} G_1(\Delta, \ell) \Big] \quad (13)$$

where $G_1(\Delta, \ell) \triangleq 1$ for $\ell = 0$, and for $\ell > 0$, it follows

$$G_1(\Delta, \ell) = \sum_{q=1}^{m} \Big[\mathcal{C}_{m-q}^{m} (\Pr\{a \leq \Delta\})^{m-q} \times \frac{1}{q^\ell} \times \mathcal{F}^{(q)}(\Delta, \ell, q) \Big], \quad (14)$$

in which $\mathcal{F}^{(i)}(\Delta, \ell, q)$ stands for the sum of the outage probabilities of ℓ consecutive retransmissions over all possible permutations of ℓ forwarding relays chosen independently each time from the relays $r_1, \ldots, r_i$ in $\mathcal{Q}$ with $|\mathcal{Q}| = q$. For $q \geq 2$ and $\ell > 0$, $\mathcal{F}^{(q)}(\Delta, \ell, q)$ can be recursively expressed as

$$\mathcal{F}^{(i)}(\Delta, \zeta_i, q) = \sum_{\zeta_{i-1}=0}^{\zeta_i} \mathcal{C}_{\zeta_{i-1}}^{\zeta_i} (e^{-\frac{\Delta}{\rho\beta_1}})^{\mu(\zeta_i, \zeta_{i-1})} \times F(\Delta, (\zeta_i - \zeta_{i-1})q) \times \mathcal{F}^{(i-1)}(\Delta, \zeta_{i-1}, q) \quad (15)$$

with $\mathcal{F}^{(2)}(\Delta, \zeta_2, q) \triangleq \sum_{\zeta_1=0}^{\zeta_2} \mathcal{C}_{\zeta_1}^{\zeta_2} (e^{-\frac{\Delta}{\rho\beta_1}})^{\mu(\zeta_2, \zeta_1)} \times F(\Delta, q\zeta_1) \times F(\Delta, (\zeta_2 - \zeta_1)q)$, where we define $\zeta_q \triangleq \ell$ for the case of $i = q$, and $\mu(\zeta_i, \zeta_{i-1}) \triangleq \delta_f[\zeta_i - \zeta_{i-1}] + \delta_f[\zeta_{i-1}] - \delta_f[\zeta_i + \zeta_{i-1}]$ such that $\mu(\zeta_i, \zeta_{i-1}) = 1$ if $\zeta_{i-1} = 0$ or $\zeta_{i-1} = \zeta_i$ where $\delta_f[\cdot]$ is the delta function; otherwise, $\mu(\zeta_i, \zeta_{i-1}) = 0$. As for $q = 1$, we have $\mathcal{F}^{(1)}(\Delta, \ell, 1) \triangleq F(\Delta, \ell)$.

Proof: In (13), $(\Pr\{a \leq \Delta\}^m \Pr\{w < \delta\})^{n-\ell}$ results from the cases of $\mathcal{Q} = \emptyset$ with $n - \ell$ retransmissions by the source, and $G_1(\Delta, \ell)$ further calculates the performance of the following ℓ ARQ rounds done by relays selected from $\mathcal{Q}$ given that $\mathcal{Q} \neq \emptyset$. More specifically, the performances of the SOAF-A ARQ scheme with different $|\mathcal{Q}| = q \in \mathcal{I}_1^m$ are characterized by (14). See Appendix C for the details. ∎

By replacing $F(\Delta, \ell)$ in (15) with its lower bound $\widetilde{F}(\Delta, \ell)$, we can have a lower bound for $\mathcal{F}^{(q)}(\Delta, \ell, q)$, which is given by $q^\ell \Pr\{a > \Delta\}^q \Pr\{b < \delta\}^{q \times \ell}$. Substituting this result back into (14), a lower bound for $\mathcal{P}_{\text{out},n}^{\text{SOAF-A}}$ can thus be obtained:

Corollary 1:

$$\widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-A}} = \Pr\{w < \delta\} \times \sum_{\ell=0}^{n} \Big[(\Pr\{a \leq \Delta\}^m \Pr\{w < \delta\})^{n-\ell} \widetilde{G}_1(\Delta, \ell) \Big] \quad (16)$$

where $\widetilde{G}_1(\Delta, \ell) \triangleq \sum_{q=1}^{m} \mathcal{C}_{m-q}^{m} \Pr\{a \leq \Delta\}^{m-q} \Pr\{a > \Delta\}^q \Pr\{b < \delta\}^{q \times \ell}$ with $\widetilde{G}_1(\Delta, 0) \triangleq 1$.

Proof: See Appendix D. ∎

According to Proposition 1, different thresholds for ARQ-SAF result in different outage probabilities or even diversity losses. Similarly, let $\Delta := \lambda\delta$ for the SOAF-A ARQ scheme. Given $\lambda \in \mathbb{R}^+$, the diversity order of $\mathcal{P}_{\text{out},n}^{\text{SOAF-A}}$ can be analyzed by Lemma 2. If $\Delta > \delta$, it follows that $\mathcal{P}_{\text{out},n}^{\text{SOAF-A}} \doteq \widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-A}}$ since $F(\Delta, \ell) \doteq \widetilde{F}(\Delta, \ell)$. In this case, by (16), we thus obtain

$$\mathcal{P}_{\text{out},n}^{\text{SOAF-A}} \stackrel{d}{=} \frac{1}{\rho} \sum_{\ell=0}^{n} \left[\left(\frac{1}{\rho^m} \frac{1}{\rho} \right)^{n-\ell} \left(\sum_{q=1}^{m} \frac{1}{\rho^{m-q}} \frac{1}{\rho^{q \times \ell}} \right)^{1-\delta_f[\ell]} \right] \quad (17)$$

where $\delta_f[\cdot]$ is the delta function. The term $(\frac{1}{\rho^m} \frac{1}{\rho})^{n-\ell}$ in (17) results from $n - \ell$ retransmissions by the source, which means the diversity order will increase by $m + 1$ with every round of the $n - \ell$ ARQs with $|\mathcal{Q}| = 0$. In comparison with the cases of $|\mathcal{Q}| = q \geq 1$ in (17), the diversity order offered by each round of ARQs through relaying only increases by m at most. As a result, at high SNR, for $n \geq 1$, the $\mathcal{P}_{\text{out},n}^{\text{SOAF-A}}$ will be dominated by the case of $|\mathcal{Q}| \neq 0$, i.e., when $\ell = n$ in (17), leading to

$$\mathcal{P}_{\text{out},n}^{\text{SOAF-A}} \stackrel{d}{=} \frac{1}{\rho} \times \sum_{q=1}^{m} \frac{1}{\rho^{m+q(n-1)}}. \quad (18)$$

By (18), the diversity order of $\mathcal{P}_{\text{out},n}^{\text{SOAF-A}}$ for $n = 1$ is equal to $m + 1$ regardless of q. For $n \geq 2$, $\mathcal{P}_{\text{out},n}^{\text{SOAF-A}}$ is dominated by the case of $q = 1$, i.e. $|\mathcal{Q}| = 1$. As a result, the diversity order of $\mathcal{P}_{\text{out},n}^{\text{SOAF-A}}$ only increases with n and is equal to $m + n$ for $n \geq 1$.

The results can be verified with the outage probabilities presented in Fig. 4. Although only R-D link qualities, $\rho|h_{j,rd}|^2$, are used for relay selection in SOAF-A, the SOAF relaying scheme is able to exploit the temporal diversity through ARQs if $\Delta > \delta$. Nevertheless, the diversity order only increases by 1 in each round after the first ARQ round.

On the other hand, if $\Delta < \delta$, the diversity order is limited to two due to the poor S-R link qualities and the selection rule of SOAF. We leave the proof in Appendix E. In comparison, the diversity order of ARQ-OAF is equal to $m + 1$ by Proposition 2, but both $\rho|h_{j,sr}|^2$ and $\rho|h_{j,rd}|^2$ are required for the destination to choose the relay according to (9).

C. ARQ with the Type B of SOAF Relaying (SOAF-B)

Based on the previous diversity analysis for SOAF-A ARQ, the key to further improve the diversity via ARQs is to increase the cardinality of $\mathcal{Q}$, i.e., $|\mathcal{Q}|$, through ARQs as well. This cannot be made possible without the unqualified relays being able to continue overhearing the signals forwarded by relays in $\mathcal{Q}$ during the process of ARQs. If proper conditions can be set on the link qualities, $\rho|h_{i,j}|^2$, between the transmitting and receiving relays to qualify and bring new relays into $\mathcal{Q}$, then the diversity may no longer be limited by the case of $|\mathcal{Q}| = 1$. This type of the SOAF scheme is referred to as the SOAF-B ARQ protocol. The functioning of the protocol is illustrated in Fig. 5. For convenience, we define a random variable $c \triangleq \rho|h_{i,j}|^2 \sim \text{Exp}(\rho\beta_3)$ for the R-R link quality.

As shown in this figure, the active relay, R5, of $\mathcal{Q}$ received a signal from R3 in the previous ARQ and is currently forwarding

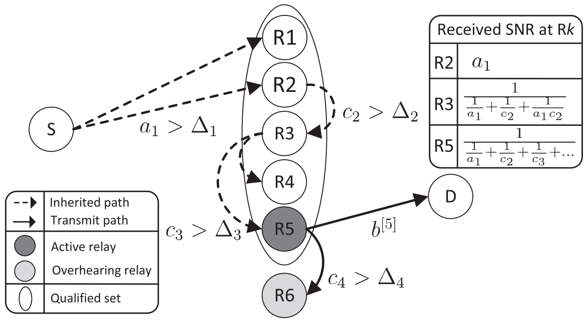


Fig. 5. An illustration for ARQs with SOAF-B relaying ($m = 6$). The subscript of c_i is used to indicate the number of hops before reaching the destination, and Δ_i is the threshold for the link quality of the i-th hop. In addition, $b^{[5]}$ represents the highest $\rho|h_{j,rd}|^2$ of relays in $\mathcal{Q}$ with $|\mathcal{Q}| = 5$.

the signal to the destination. The relay R6 in the complement set of $\mathcal{Q}$, denoted by $\mathcal{Q}^c$, overhears the signal from R5. If $c_4 = \rho|h_{5,6}|^2$ exceeds a threshold, say Δ_4 with 4 indicating the number of hops before reaching the destination, then R6 will be taken out of the set $\mathcal{Q}^c$ and put into the qualified set $\mathcal{Q}$. In the next round of ARQ, if any, the destination then chooses the relay with the highest $\rho|h_{j,rd}|^2$ from the new set $\mathcal{Q}$ to forward the signal, even if the signal from R6 has accumulated more noise through the hops from the source to R2, then R3 and then R5.

We define a threshold for each hop to control the channel quality of the entire relaying path. Since the maximal number of hops is limited to $\min[m, N]$, we thus have an array of thresholds, $\overline{\Delta} \triangleq [\Delta_1, \ldots, \Delta_i, \ldots, \Delta_{\min[m,N]}]$ with Δ_i corresponding to the threshold for the link quality of the i-th hop. In general, for an active relay that forwards a signal which has already gone through k hops, the instantaneous received SNR at the destination is given by [27]

$$\mathrm{SNR}_{\mathrm{inst},rd} = \left[\left(1 + \frac{1}{a_1}\right) \prod_{i=2}^{k} \left(1 + \frac{1}{c_i}\right) \left(1 + \frac{1}{b^{[q]}}\right) - 1\right]^{-1} \quad (19)$$

where $b^{[q]} = \max_{j\in\mathcal{Q}} \rho|h_{j,rd}|^2$ given $|\mathcal{Q}| = q$.

We recall from Proposition 1 that the potential diversity of ARQ-SAF can be achieved if the thresholding mechanism prevents low-quality signals from being relayed, i.e., let $\Delta := \lambda\delta > \delta$. Thus, to exploit the diversity in SOAF-B ARQs, we also need to define a requirement for relays in different hops to qualify their received instantaneous SNR ($\mathrm{SNR}_{\mathrm{inst}}$). By the form of (19), we may define the requirement as follows:

Requirement 1: For a k-hop signal received by a relay in $\mathcal{Q}$, the received SNR of the relay satisfies $\mathrm{SNR}_{\mathrm{inst}} = [(1 + \frac{1}{a_1}) \prod_{i=2}^{k} (1 + \frac{1}{c_i}) - 1]^{-1} \geq \underline{\lambda}\delta$, given $\underline{\lambda} > 1$.

Under Requirement 1, the capacity outage probability corresponding to (19) becomes

$$\begin{aligned}\Pr\{\mathrm{SNR}_{\mathrm{inst},rd} < \delta\} &\leq \Pr\left\{\left(1 + \frac{1}{\underline{\lambda}\delta}\right)\left(1 + \frac{1}{b^{[q]}}\right) > 1 + \frac{1}{\delta}\right\} \\ &= \left(\Pr\left\{b < \frac{1}{\frac{1+1/\delta}{1+1/(\underline{\lambda}\delta)} - 1}\right\}\right)^q \quad (20)\end{aligned}$$

where $\frac{1+1/\delta}{1+1/(\underline{\lambda}\delta)} > 1$ since $\underline{\lambda} > 1$, and note that $\Pr\{b^{[q]} < \epsilon\} = \Pr\{b < \epsilon\}^q$ holds true if $\epsilon > 0$ and remains fixed with ρ. In other words, this probability can attain the diversity order q. Therefore, if there is a qualified set $\mathcal{Q}$ with $|\mathcal{Q}| = q > 0$, and every relay chosen from $\mathcal{Q}$ for ARQs satisfies Requirement 1, then the diversity order offered by relaying can increase by q for every extra ARQ round according to (20).

Following the above requirement and the result of (20), we finally arrive at a theorem for the SOAF-B relaying method to exploit the potential diversity of ARQs.

Theorem 1: Given $\mathcal{R}$, if the thresholds of $\overline{\Delta}$ are constant with ρ but sufficiently large for Requirement 1 to be satisfied, then the diversity order of the capacity outage probability after n rounds of SOAF-B ARQs is given by $(m \times n + 1)$.

Proof: Starting with ARQ1, we first partition the outage probability of SOAF ARQ1 into two parts with one of them corresponding to the event of $|\mathcal{Q}| = 0$, and the other for $|\mathcal{Q}| \neq 0$. The first part of this probability is given by $\Pr\{a < \Delta_1\}^m \Pr\{w < \delta\}$ whose diversity order can attain $m + 1$ since $\Pr\{w < \delta\} \overset{d}{=} \frac{1}{\rho}$ and $\Pr\{a < \Delta_1\}^m = (1 - e^{-\frac{\Delta_1}{\rho\beta_1}})^m \overset{d}{=} \frac{1}{\rho^m}$ under the condition that the threshold Δ_1 is constant with ρ. In comparison, the second part attains the diversity order of m only, since given $q = |\mathcal{Q}| > 0, \Pr\{a < \Delta_1\}^{m-q} \overset{d}{=} \frac{1}{\rho^{m-q}}$ and the retransmission by a relay chosen from $\mathcal{Q}$ for ARQ1 offers an extra diversity order of q according to (20). As a result, the performance of the SOAF-B scheme will be dominated at high SNR by the case of $q > 0$. Thus, the diversity order of the outage probability after ARQ1 will achieve a total of $m + 1$, with one diversity offered by ARQ0.

Similarly, the diversity order after ARQ2 will increase by m since the probability that $|\mathcal{Q}|$ becomes q' after ARQ1 can achieve the order of $\frac{1}{\rho^{m-q'}}$ due to $\Pr\{c < \Delta_2\}^{m-q'} \overset{d}{=} \frac{1}{\rho^{m-q'}}$, and the relaying in ARQ2 will contribute a diversity order of q' according to (20). Following the same argument for the subsequent ARQs, we can conclude that the diversity order will increase by m for every extra ARQ round. As a result, the outage probability after n rounds of SOAF-B ARQs can attain the diversity order of $m \times n + 1$. ■

We note that Requirement 1 also applies to the SOAF-A ARQ scheme (ARQ-SOAF-A) where relays essentially overhear the signals from the source only, namely, only the S-R hops apply in Requirement 1. As such, we have $a_1 \geq \underline{\lambda}\delta > \delta$, and hence $a_1 > \Delta_1 \geq \underline{\lambda}\delta > \delta$, which matches the threshold setting for SOAF-A relaying in Section IV-B.

Remarks: The SOAF-A relaying method can be used in typical two-hop AF relaying networks, and only requires the R-D channel state information for opportunistic relaying. Besides, the notion of SOAF-B relaying can also be applied to multi-hop wireless networks. In contrast to the typical sequential relaying manner [10], [11], the SOAF-B protocol allows a source packet to go through a dynamic relaying path before reaching the destination, and is thus able to exploit more spatial diversities. For instance, given k relays and $k + 1$ hops, namely $m = k$ and $N = k$, the SOAF-B protocol can provide a diversity order of $k^2 + 1$ which is greater than the diversity order of $k + 1$ of the multi-hop scheme in [10].

D. Lower Bound of the Outage Probability of SOAF-B ARQ

In contract to ARQ-SOAF-A, the exact outage probability of the SOAF-B ARQ scheme, $\mathcal{P}_{\text{out},n}^{\text{SOAF-B}}$, is much more difficult to analyze since there are too many possible inherited relationships from the source to the final forwarding relay. The outage events of retransmissions will become correlated once the relayed signals had ever commenced from a parent relay. Instead of directly tackling on the exact outage probability, we derive an analytic lower bound for $\mathcal{P}_{\text{out},n}^{\text{SOAF-B}}$, denoted by $\widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-B}}$, whose derivation is provided below. The tightness of this lower bound will be verified later in Fig. 8.

We present two steps to estimate the lower bound $\widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-B}}$ by ignoring the effect of noise enhancement on the forwarded signals from relays. To begin with, we use $\mathcal{Q}_\ell$ with $\mathcal{Q}_0 = \{\emptyset\}$ to stand for the qualified set at the beginning of ARQℓ. Then, we divide $\mathcal{Q}_\ell$ into $\min[\ell, m]$ subsets, denoted by $\underline{Q}_{\ell,k}$ for $k = 1, \ldots, \min[\ell, m]$, where $\underline{Q}_{\ell,k}$ only contains relays that receive k-hop signals in $\mathcal{Q}_\ell$. Finally, we use $\underline{q}_{\ell,k}$ to denote the number of relays *newly* brought into $\underline{Q}_{\ell,k}$ at the end of ARQ$\ell - 1$, i.e. $\underline{q}_{\ell,k} \triangleq |\underline{Q}_{\ell,k} \backslash \underline{Q}_{\ell-1,k}|$. As an example in Fig. 6, the subset $\underline{Q}_{4,3}$ has contained all the relays in $\underline{Q}_{3,3}$, and can be further enlarged at the end of ARQ3 if the active relay for ARQ3 is chosen from $\underline{Q}_{3,2}$ and there are overhearing relays (in $\mathcal{Q}^c$) to be added into $\mathcal{Q}_4$ since their corresponding channel qualities from the active relay exceed Δ_3. The number of the relays newly added into $\underline{Q}_{4,3}$ is, therefore, $\underline{q}_{4,3}$.

The two steps to find $\widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-B}}$ are stated below:

Step 1: Set the variables $\underline{q}_{\ell,k} = 0$ for $\ell \in \mathcal{I}_1^n$ and $k \in \mathcal{I}_1^{\min[\ell,m]}$, whereby we define $|\mathcal{Q}_\ell| \triangleq \sum_{k=1}^{\min[\ell,m]} \sum_{i=k}^{\ell} \underline{q}_{i,k}$ and $|\underline{Q}_{\ell,k}| \triangleq \sum_{i=k}^{\ell} \underline{q}_{i,k}$.

Step 2: Follow the formula:

$$\widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-B}} \triangleq \Pr\{w < \delta\} \times \sum_{\ell=0}^{n} \Big[(\Pr\{a_1 \le \Delta_1\}^m \Pr\{w < \delta\})^{n-\ell}\, \widetilde{G}_2(\overline{\Delta}, \ell) \Big] \tag{21}$$

where $\widetilde{G}_2(\overline{\Delta}, \ell) \triangleq 1$ for $\ell = 0$, and for $\ell \in \mathcal{I}_1^n$, it follows

$$\widetilde{G}_2(\overline{\Delta}, \ell) \triangleq \sum_{\underline{q}_{1,1}=1}^{m} F_{1,1}(\underline{q}_{1,1}) \times \sum_{\underline{q}_{2,2}=0}^{m-|\mathcal{Q}_1|} F_{2,2}(\underline{q}_{2,2}) \times \left(\frac{|\underline{Q}_{2,1}|}{|\mathcal{Q}_2|} \sum_{\underline{q}_{3,2}=0}^{m-|\mathcal{Q}_2|} F_{3,2}(\underline{q}_{3,2}) + \frac{|\underline{Q}_{2,2}|}{|\mathcal{Q}_2|} \sum_{\underline{q}_{3,3}=0}^{m-|\mathcal{Q}_2|} F_{3,3}(\underline{q}_{3,3}) \right) \times \cdots \times \underbrace{\left(\sum_{k=2}^{\min[\ell,m]} \frac{|\underline{Q}_{\ell-1,k-1}|}{|\mathcal{Q}_{\ell-1}|} \sum_{\underline{q}_{\ell,k}=0}^{m-|\mathcal{Q}_{\ell-1}|} F_{\ell,k}(\underline{q}_{\ell,k}) \right)}_{\text{The expressions for possible } |\mathcal{Q}| \text{ at ARQ}\ell}, \tag{22}$$

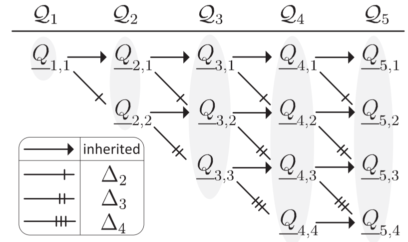


Fig. 6. An illustration for the evolution of $\mathcal{Q}$ in 5 ARQs in a system that uses 4 relays for the SOAF-B relaying scheme. In the figure, the arrow represents $\underline{Q}_{\ell,k} \subseteq \underline{Q}_{\ell+1,k}$, and the line between $\underline{Q}_{\ell,k}$ and $\underline{Q}_{\ell+1,k+1}$ shows that for a relay chosen from $\underline{Q}_{\ell,k}$ for ARQℓ, the overhearing relays in $\mathcal{Q}^c$ will check link quality from this active relay against the threshold Δ_{k+1} to see if they can join $\underline{Q}_{\ell+1,k+1}$ or not.

with $F_{1,1}(q)$ and $F_{i,k}(q)$ for $i, k \ge 2$, given by $F_{1,1}(q) = \mathcal{C}_q^m \Pr\{a \le \Delta_1\}^{m-q} \Pr\{a > \Delta_1\}^q \Pr\{b < \delta\}^q$, and

$$F_{i,k}(q) = \mathcal{C}_q^{m-|\mathcal{Q}_{i-1}|} \Pr\{c \le \Delta_k\}^{m-(|\mathcal{Q}_{i-1}|+q)} \times \Pr\{c > \Delta_k\}^q \Pr\{b < \delta\}^{(|\mathcal{Q}_{i-1}|+q)}. \tag{23}$$

Here, we note that the variables $\underline{q}_{\ell,k}$, initiated in Step 1, will be updated with the corresponding summation index in (22). Basically, the formula of $\widetilde{G}_2(\overline{\Delta}, \ell)$ follows the evolution of $\mathcal{Q}$ in ℓ ARQ rounds, as illustrated with Fig. 6. And in (22), the term $\frac{|\underline{Q}_{\ell-1,k-1}|}{|\mathcal{Q}_{\ell-1}|}$ stands for the probability of a partial event that the relay for ARQ$\ell - 1$ is chosen from $\underline{Q}_{\ell-1,k-1}$. Conditioned on this event, $F_{\ell,k}(\underline{q}_{\ell,k})$ further characterizes the effect of $\underline{q}_{\ell,k}$ relays newly brought into $\mathcal{Q}$ on the performance of ARQℓ. If all the thresholds in $\overline{\Delta}$ are set constant with ρ, it then follows that $F_{1,1}(q) \overset{d}{=} F_{i,k}(q) \overset{d}{=} \frac{1}{\rho^m}$, leading to $\widetilde{G}_2(\overline{\Delta}, \ell) \overset{d}{=} \frac{1}{\rho^{m \times \ell}}$. We can thus obtain $\widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-B}} \overset{d}{=} \frac{1}{\rho^{m\,n+1}}$.

V. Threshold Assignment Methods

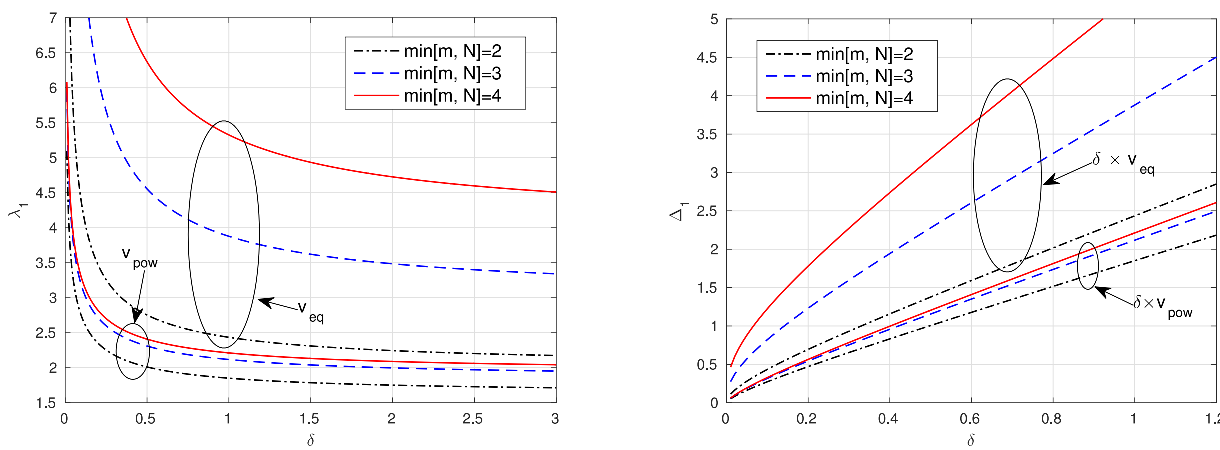
As a short summary in Table I, for ARQ-SAF and ARQ-SOAF-A, their Δ's can simply be set as $\lambda\delta$ with $\lambda > 1$ in order to exploit their potential diversities. In addition to diversity property, to obtain a better SNR gain, the λ's in fact cannot be set too high or too low. If the parameters λ's are set too low, e.g., close to 1, there will be more noise or deeper channel fading coupled in the relayed signals, while the λ's too high will also result in performance losses due to a higher probability of using poor direct S-D links.

According to our experiences from simulations, we suggest to set $\lambda = 1.5$ for ARQ-SAF and ARQ-SOAF-A. As for the SOAF-B ARQ scheme (ARQ-SOAF-B), we can first let $\overline{\Delta} := \delta[\lambda_1, \ldots, \lambda_{\min[m,N]}]$ with $\lambda_i \in \mathbb{R}^+, \forall i$. Followed, to attain the diversity order stated in Theorem 1, by Requirement 1, given $\underline{\lambda} > 1$, we need to find proper settings for λ_i such that

$$\left(1 + \frac{1}{\lambda_1 \delta}\right) \times \cdots \times \left(1 + \frac{1}{\lambda_{\min[m,N]}\delta}\right) \le 1 + \frac{1}{\underline{\lambda}\delta}. \tag{24}$$

TABLE I
A SUMMARY ON THE DIVERSITY ORDERS OF DIFFERENT ARQ SCHEMES AFTER ARQn

ARQ scheme	SAF		SOAF-A		SOAF-B	AF	OAF
Threshold(s) rule	$\Delta < \delta$	$\Delta > \delta$	$\Delta < \delta$	$\Delta > \delta$	(25) or (26)	-	-
Diversity order	2	$n+1$	2	$m+n$	$m \times n + 1$	2	$m+1$



(a) (b)

Fig. 7. The solutions, v_{eq} and v_{pow}, of (25) and (26) with different δ and $\min[m, N]$, given $\underline{\lambda} = 1.01$. (a) λ_1 versus δ, (b) Δ_1 versus δ.

This is because given $a_1 > \Delta_1 = \lambda_1\delta$ and $c_i > \Delta_i = \lambda_i\delta, \forall i \in \mathcal{I}_2^k$, the condition (24) will imply $(1+\frac{1}{a_1})\prod_{i=2}^k(1+\frac{1}{c_i}) < (1+\frac{1}{\lambda_1\delta})\prod_{i=2}^k(1+\frac{1}{\lambda_i\delta}) \le 1+\frac{1}{\underline{\lambda}\delta}, \forall k \in \mathcal{I}_1^{\min[m,N]}$.

Based on (24), we study in this paper two intuitive ways to set the thresholds, referred to as the equal threshold method and the power-law threshold method. For the first one, the thresholds for each hop are assumed equal, namely $\lambda_1 = \cdots = \lambda_{\min[m,N]} \triangleq v_{\text{eq}}$. Applying the equality to (24), we have the closed-form expression of v_{eq}, given by

$$v_{\text{eq}} = \frac{1}{-1+(1+\frac{1}{\underline{\lambda}\delta})^{\frac{1}{\min[m,N]}}} \times \frac{1}{\delta}. \qquad (25)$$

From (25), we can see that $\lambda_i = v_{\text{eq}}$ varies with δ and $\min[m, N]$, namely the dimension of $\overline{\Delta}$, and decreases as $\delta = 2^{\mathcal{R}} - 1$ increases. This makes the setting of λ_i different from the constant setting of λ for ARQ-SAF and ARQ-SOAF-A. For a lower source rate $\mathcal{R}$, the ratio $\lambda_i = \Delta_i/\delta$ should be set higher for the ARQ-SOAF-B scheme.

Though simple and informative, the equal setting on Δ_i makes the thresholds for the early hops of the forwarded signals unnecessarily too high, given that the thresholds are supposed to be higher as the number of hops increases in order to combat the noises carried along the hops. To overcome this problem, we may set $\lambda_i = v_{\text{pow}}^i$ with $v_{\text{pow}} > 1$ to make λ_i gradually increase with i. This method is referred to as the power-law method. Substituting this setting into (24), we have

$$\left(1+\frac{1}{v_{\text{pow}}\delta}\right)\left(1+\frac{1}{v_{\text{pow}}^2\delta}\right)\times\cdots\times\left(1+\frac{1}{v_{\text{pow}}^{\min[m,N]}\delta}\right) = \left(1+\frac{1}{\underline{\lambda}\delta}\right). \qquad (26)$$

By the Bisection method, the parameter v_{pow} can be solved.

Fig. 7(a) shows the results of λ_1, which equal v_{eq} and v_{pow} in the two threshold setting methods, respectively. The corresponding thresholds of $\Delta_1 = \delta\lambda_1$ are drawn in Fig. 7(b). As expected, v_{eq} is higher than v_{pow}, and both increase with $\min[m, N]$. In addition, the values of v_{eq} and v_{pow} converge rapidly w.r.t. δ when δ is above a certain value, e.g., $\delta > 1$.

The outage probability of ARQ-SOAF-B with different threshold assignment methods are presented in Fig. 8 for the case of $m = 3$ and $N = 3$. The thresholds by the methods of (25) and (26) are obtained with $\underline{\lambda} = 1.01$ to satisfy Requirement 1. As shown in the figures, $\mathcal{P}_{\text{out},n}^{\text{SOAF-B}}$ with the methods of (25) and (26) attain the full diversity, while $\mathcal{P}_{\text{out},n}^{\text{SOAF-B}}$ with $\overline{\Delta} := \delta[2, 4, 8]$ may lose its diversity if $\delta \le 1.97$ or $\mathcal{R} \le 1.57$, according to (24). The $\mathcal{P}_{\text{out},n}^{\text{SOAF-B}}$ for another setting of $\overline{\Delta} = \delta[1.6, 1.6^2, 1.6^3]$ is shown in Fig. 8(b). In contract to $\mathcal{P}_{\text{out},n}^{\text{SOAF-B}}$ for $\overline{\Delta} = \delta[2, 2^2, 2^3]$, the outage probability in this case loses its diversity since $1.6 < v_{\text{pow}} = 1.89$ which is obtained with (26) for $\mathcal{R} = 3$. Also shown in the figures are the analytic lower bounds $\widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-B}}$ obtained with the threshold setting of v_{pow} for (21) to (23). The results show that $\widetilde{\mathcal{P}}_{\text{out},n}^{\text{SOAF-B}}$ is a tight bound and attains the full diversity.

For performance assessment, in Fig. 9, we also present the results of the DF counterparts of the proposed SOAF ARQ schemes, denoted by SODF-A and SODF-B, respectively, to serve as the performance benchmarks. The difference between SODF and SOAF is that DF schemes do decoding at relays, and only relays that succeed in decoding are brought into $\mathcal{Q}$. As can be seen in this figure, the outage probabilities of the ARQ-SOAF-A can be shown very close to those of the ARQ-SODF-A, even though there is no decoding at relays. In addition, the SOAF-B ARQ scheme

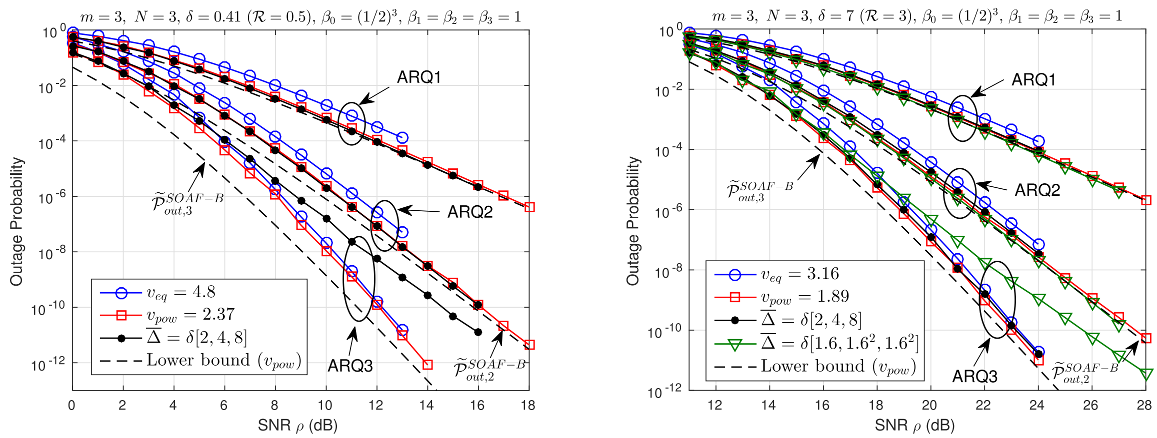


Fig. 8. Outage probabilities of the ARQ-SOAF-B scheme with different threshold assignments. (a) The case of $\mathcal{R} = 1/2$ ($\delta = 0.41$), (b) The case of $\mathcal{R} = 3$ ($\delta = 7$).

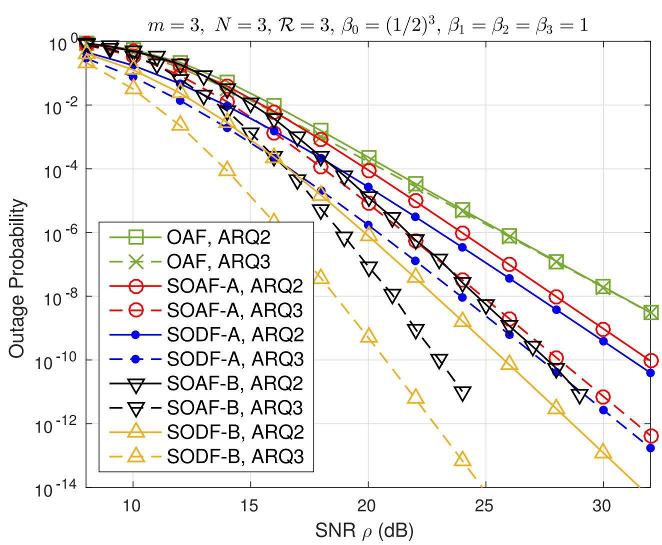


Fig. 9. Performance comparisons between the proposed ARQ schemes and their DF counterparts in the case of $\mathcal{R} = 3$, where we set $v_{\text{pow}} = 1.86$ for SOAF-B and $\lambda = 1.5$ for SOAF-A.

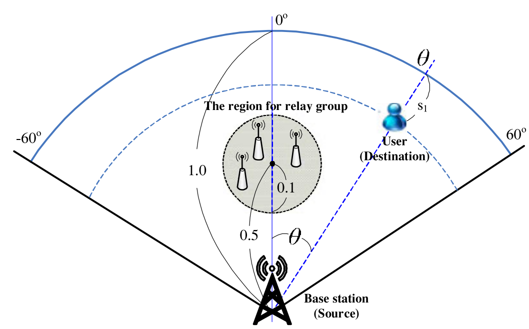


Fig. 10. A cell sector assisted with relays distributed in a limited region.

also provides a performance only 2 dB inferior to that of the SODF-B.

VI. Simulations Studies in Throughput

The outage probability analyses in the previous sections have presented the potential of the proposed SOAF relaying methods to enhance the reliability of ARQs. The reliability in fact can be traded off for throughput in wireless transmissions. In this section, we study by simulations the throughput performance of the proposed ARQ protocols. The results demonstrate their effectiveness in throughput improvement, in particular for users close to cell boundaries.

A. Simulation Settings

We consider a cell that has 3 sectors, and focus on one of the sectors in our simulations. The sector is assumed to use an ARQ scheme that employs a total of m relays to perform delay-aware data retransmissions, with the maximum number of ARQ, N, equal to 3. To fairly serve the users distributed in the sector, and meanwhile to avoid the interference to relays in adjacent sectors, the m relays are considered deployed within a limited circular area around the sector center, as depicted in Fig. 10. We also consider that users at sector edge will be protected with the typical interference avoidance method [29] to avoid the inter-cell (sector) interference. We thus evaluate the performances of using different ARQ schemes in a sector without particularly considering the co-channel interference.

On the other hand, for convenience, the cell coverage radius is normalized to one. Given the distance between any two nodes, the corresponding channel variance is defined as the inverse of the distance raised to the power of the path loss exponent which, in the simulations, is set equal to 3. The channel variance for any transmit-and-receive pair in Fig. 10 can thus be obtained according to their geometric relations. As for threshold assignments, we define $\Delta = 1.5\delta$ for ARQ-SAF and ARQ-SOAF-A, and adopt the power-law threshold setting method in Section V for ARQ-SOAF-B, given that $\underline{\lambda} = 1.01$.

Based on the above settings, we simulate the throughput of different AF-relay-assisted ARQ schemes. In the simulations, a packet will be dropped only if it fails to be delivered to the destination after ARQN. Given a target P_t, our rate adaptation

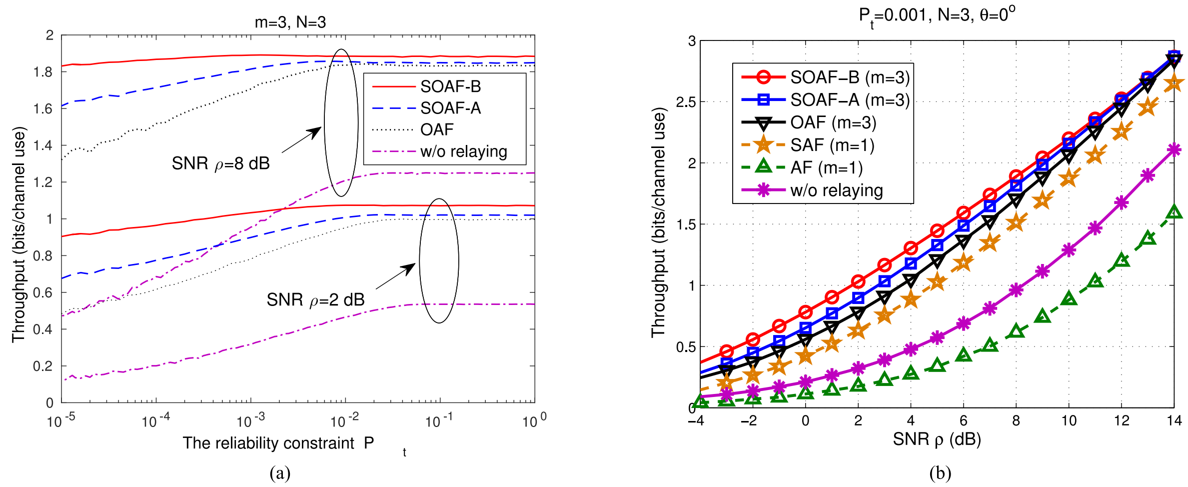


Fig. 11. Throughputs $\mathcal{T}_{P_t}$ of different ARQ schemes for a user located at $\theta = 0^\circ$ and $s_1 = 0$. (a) Throughput $\mathcal{T}_{P_t}$ versus P_t, (b) Throughput $\mathcal{T}_{P_t}$ versus SNR ρ.

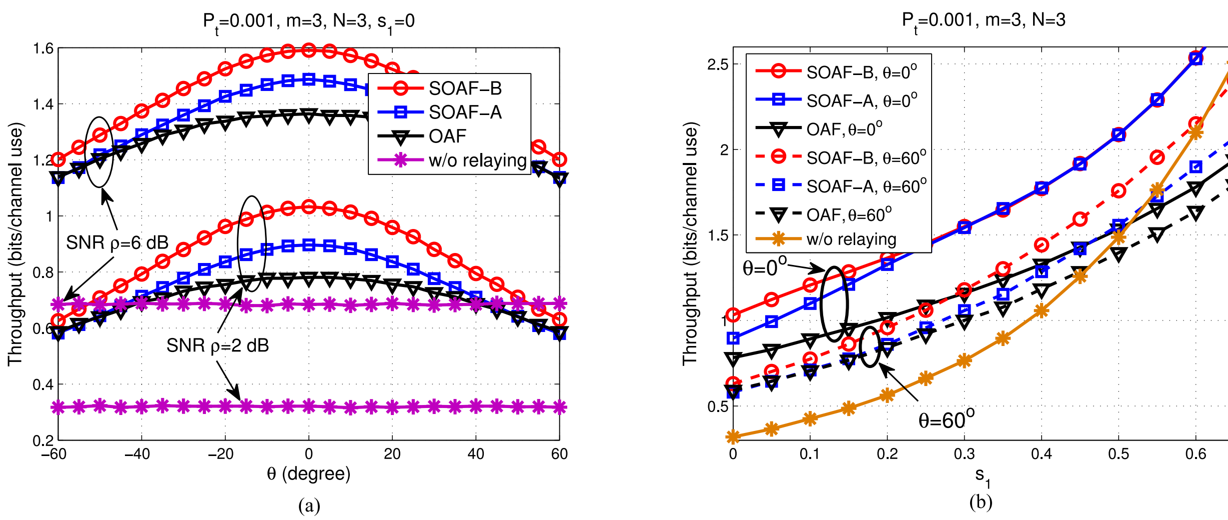


Fig. 12. The throughput $\mathcal{T}_{P_t}$ versus different θ and s_1. (a) $\mathcal{T}_{P_t}$ versus θ ($s_1 = 0$), (b) $\mathcal{T}_{P_t}$ versus s_1.

strategy is subject to a reliability constraint of $\mathcal{P}_{\text{out},N} \leq P_t$. Due to the fact that different relays deployments in the circular area may lead to different throughputs, to properly evaluate the system performance, we thus assume that the employed relays are randomly located in this area, and define our throughput metric as follows to take the randomness into account:

$$\mathcal{T}_{P_t} \triangleq \mathop{\mathbb{E}}_{\text{Relays' locations}} \left[\max_{\mathcal{R}\geq 0,\ \mathcal{P}_{\text{out},N} \leq P_t} \frac{\mathcal{R}(1-\mathcal{P}_{\text{out},N})}{N\mathcal{P}_{\text{out},N-1} + \sum_{\ell=1}^{N-1} \ell(\mathcal{P}_{\text{out},\ell-1} - \mathcal{P}_{\text{out},\ell})} \right] \tag{27}$$

where the objective term is the long-term average throughput derived by the renewal-reward theorem (see [21] for details).

B. The Throughput Performances

In Fig. 11(a), we present the tradeoff between $\mathcal{T}_{P_t}$ and the reliability constraint P_t. When a high reliable transmission is required, the two SOAF ARQ schemes show more advantages in throughput than the OAF and the non-relaying schemes. In particular, having fully exploited the temporal and spatial diversities, the SOAF-B ARQ scheme trades off less throughput to attain the target reliability P_t in comparison with the others.

In Fig. 11(b), we compare the throughputs of the different ARQ schemes versus the SNR ρ when $P_t = 10^{-3}$ and the user is fixed at $\theta = 0^\circ$ and $s_1 = 0$. We find that the ARQ schemes with OR outperform the non-relaying one, even thought ARQ-OAF in fact suffers from severe diversity losses. Although the throughput of SOAF-A is slightly worse than that of SOAF-B, its performance is in fact pronounced considering its much simpler mechanism for relaying. We also notice that the single-relay SAF ARQ scheme can effectively compensate the path loss of signal power in retransmissions and, thus, benefits greatly in throughput enhancement. In comparison, the performance of the ARQ-AF scheme degrades significantly due to the lack of a signal pre-screening process at the relay.

The next two studies are considered to examine the effects of different user locations on throughput. In Fig. 12(a), we assume that the user moves along the cell edge, i.e. we set $s_1 = 0$ while vary θ from -60° to 60°. Given $\rho = 2$ dB, at $\theta = 0^\circ$, the throughput of the ARQ-SOAF-B is around 3.2 times higher than the non-relaying case, while $\mathcal{T}_{P_t}$ of the ARQ-SOAF-A is 2.8 times higher. Even at the corners of the sector, namely $\theta = 60^\circ$ or -60°, the two SOAF schemes still perform well and have their throughputs almost 2 times higher than the non-relaying case.

On the other hand, we vary s_1 from 0 to 0.6 to see the influence on throughput when the user moves towards the base station, given $\rho = 2$ dB at $s_1 = 0$. Basically, for every ARQ scheme, their throughputs will improve as s_1 increases, since the average S-D link quality is getting better. Fig. 12(b) shows that the two SOAF ARQ schemes outperform the non-relaying case when $s_1 < 0.5$. In contrast, the performance of ARQ-OAF degrades significantly as s_1 increases. The OAF scheme cannot effectively utilize the improved R-D channel gains when the user continues to move inside since its performance will be limited by the worst S-R channel qualities due to the lack of pre-screening mechanism at the relay.

VII. Conclusion

In this work, we did theoretical studies on link quality control for cooperative ARQ with opportunistic AF relaying. Our outage analysis showed that the temporal diversities of ARQs with AF relaying can be fully exploited if the channel qualities to relays exceed a proper threshold that depends on the source data rate only. By incorporating the OR mechanism into this SAF ARQ framework, two types of link quality control and ARQ schemes, the SOAF-A and SOAF-B, were developed, attempting to explore both the temporal and spatial diversities. To recover the severe diversity losses in the typical OAF ARQ scheme, the SOAF-A proposes to form a qualified set of relays before OR. Further, to exploit the full spatial diversity, the SOAF-B proposes to continuously enlarge the qualified set by allowing overhearing among relays during the process of ARQs. Analysis showed that the two proposed schemes can offer much higher diversities than the OAF ARQ if the thresholds for each hop are set properly. Feasible threshold assignment methods were then studied for the SOAF ARQ schemes to achieve their potential diversities. By simulations, the effectiveness of the proposed ARQ schemes was also presented in the throughput enhancement of cell-edge users.

Although this work provided a new look and method for quality control along each hop of multi-AF relaying systems, the ARQ analysis model has its limitations that should be addressed in future works. For instance, we consider only one source and one destination in this work. Multiple access scenarios may give rise to a user scheduling issue on relays for multiple retransmissions on different resource blocks. The outdated CSI problem is also an important and practical topic that should be further taken into account. A simpler ARQ design, like SOAF-A, may show more advantages from this perspective if having a shorter feedback delay.

Appendix A
Proof of Lemma 1

From (6), if $\Delta \geq \delta$, we have

$$\begin{aligned} F(\Delta,\ell) &= \mathop{\mathbb{E}}_{a>\Delta}\left[\left(\Pr\left\{b < \frac{a\delta+\delta}{a-\delta}\middle| a\right\}\right)^{\ell}\right] \\ &= \int_{\Delta}^{\infty}\left(1-e^{-\frac{a\delta+\delta}{\rho\beta_2(a-\delta)}}\right)^{\ell} e^{-\frac{a}{\rho\beta_1}}da \\ &= e^{-\frac{\Delta}{\rho\beta_1}} + \sum_{i=1}^{\ell}\mathcal{C}_i^{\ell}(-1)^i e^{-\frac{\delta}{\rho\beta_1}} e^{-\frac{i\delta}{\rho\beta_2}} \\ &\quad \times \Gamma\left(1, \frac{\Delta-\delta}{\rho\beta_1}; \frac{i(\delta^2+\delta)}{\rho^2\beta_1\beta_2}\right). \end{aligned} \tag{28}$$

On the other hand, for the case of $\Delta < \delta$, we have

$$\begin{aligned} F(\Delta,\ell) &= \Pr\{\delta \geq a > \Delta\} + F(\delta,\ell) \\ &\overset{(a)}{=} e^{-\frac{\Delta}{\rho\beta_1}} + \sum_{i=1}^{\ell}\mathcal{C}_i^{\ell}(-1)^i e^{-\frac{\delta}{\rho\beta_1}} e^{-\frac{i\delta}{\rho\beta_2}} \\ &\quad \times \Gamma\left(1, 0; \frac{i(\delta^2+\delta)}{\rho^2\beta_1\beta_2}\right). \end{aligned} \tag{29}$$

The equality (a) is due to $\Pr\{\delta \geq a > \Delta\} = e^{-\frac{\Delta}{\rho\beta_1}} - e^{-\frac{\delta}{\rho\beta_1}}$ and (28) with $\Delta := \delta$.

Appendix B
Proof of Lemma 2

Suppose $\Delta := \lambda\delta, \lambda > 1$. We use the value of $(\beta_1 \ln \rho)$ to partition the integral region of the random variable "a" in $F(\Delta,\ell)$. When ρ increases such that $(\beta_1 \ln \rho) > \lambda\delta$, we have

$$\begin{aligned} F(\Delta,\ell) &= \mathop{\mathbb{E}}_{\beta_1 \ln\rho > a > \lambda\delta}\left[\left(1-e^{-\frac{1}{\rho\beta_2}\frac{1+\frac{1}{a}}{\frac{1}{\delta}-\frac{1}{a}}}\right)^{\ell}\right] \\ &\quad + \mathop{\mathbb{E}}_{a\geq\beta_1\ln\rho}\left[\left(1-e^{-\frac{1}{\rho\beta_2}\frac{1+\frac{1}{a}}{\frac{1}{\delta}-\frac{1}{a}}}\right)^{\ell}\right]. \end{aligned} \tag{30}$$

The exponent $(1+\frac{1}{a})/(\frac{1}{\delta}-\frac{1}{a})$ is a decreasing function with $a > \delta$, and it will approach δ as $a \to \infty$. The first expectation term, named T_1, in (30) can thus be bounded by

$$\begin{aligned} &\left(1-e^{-\frac{\delta}{\rho\beta_2}}\right)^{\ell}\int_{\lambda\delta}^{(\beta_1\ln\rho)}\frac{1}{\rho\beta_1}e^{-\frac{a}{\rho\beta_1}}da \leq T_1 \\ &\quad \leq \left(1-e^{-\frac{\delta}{\rho\beta_2}\frac{1+\frac{1}{\lambda\delta}}{1-\frac{1}{\lambda}}}\right)^{\ell}\int_{\lambda\delta}^{(\beta_1\ln\rho)}\frac{1}{\rho\beta_1}e^{-\frac{a}{\rho\beta_1}}da \end{aligned} \tag{31}$$

where the two integrations are the same and equal to $e^{-\frac{\lambda\delta}{\rho\beta_1}} \times (1-e^{-\frac{1}{\rho}(\ln\rho-\lambda\delta/\beta_1)})$ with the same order of ρ^{-1} by definition. In other words, T_1 is of the order of $\rho^{-(\ell+1)}$.

As for the second expectation, denoted by T_2, in (30), we can similarly obtain

$$\left(1-e^{-\frac{\delta}{\rho\beta_2}}\right)^{\ell} e^{-\frac{\ln\rho}{\rho}} \leq T_2 \leq \left(1-e^{-\frac{1}{\rho\beta_2}\frac{1+\frac{1}{\beta_1\ln\rho}}{\frac{1}{\delta}-\frac{1}{\beta_1\ln\rho}}}\right)^{\ell} e^{-\frac{\ln\rho}{\rho}} \tag{32}$$

which yields $T_2 \doteq (\frac{\delta}{\rho\beta_2})^{\ell}$. As a result, we have $F(\Delta,\ell) \doteq T_2$ owing to its smaller diversity order than that of T_1. In addition, it can be easily verified from (8) that $\widetilde{F}(\Delta,\ell) \doteq (\frac{\delta}{\rho\beta_2})^{\ell}$. We thus arrive at the fact that $F(\Delta,\ell) \doteq \widetilde{F}(\Delta,\ell) \doteq (\frac{\delta}{\rho\beta_2})^{\ell}$.

As for the case of $\Delta := \lambda\delta$, $\lambda < 1$, we first define a special threshold $\Delta' := \lambda'\delta > \delta$ where $\lambda' \triangleq (1+\frac{1}{\ln\rho})$ is a function of ρ such that Δ' can be arbitrarily close to δ as $\rho \to \infty$. Using the fact that $\frac{ab}{a+b+1} \leq \min[a,b] \leq a$, and (6), we have

$$\begin{aligned}\Pr\{\Delta < a < \delta\} &\leq F(\Delta,\ell)\\ &\leq \Pr\{\Delta < a \leq \Delta'\} + F(\Delta',\ell).\end{aligned} \tag{33}$$

As a matter of fact, $F(\Delta',\ell) \overset{d}{=} \rho^{-\ell}$, and the diversity analysis can be done by replacing λ in (30) and (31) with $\lambda' = (1+1/\ln\rho)$. Specifically, we express $F(\Delta',\ell)$ in the form of (30), of which an upper bound for the first expectation term can be obtained based on (31) as

$$\begin{aligned}&\left(1-e^{-\frac{\delta}{\rho\beta_2}\frac{1+\frac{1}{\lambda'\delta}}{1-\frac{1}{\lambda'}}}\right)^{\ell}\int_{\lambda'\delta}^{(\beta_1\ln\rho)}\frac{1}{\rho\beta_1}e^{-\frac{a}{\rho\beta_1}}da\\ &=\left(1-e^{-\frac{\delta}{\rho\beta_2}\left((1+\frac{1}{\delta})\ln\rho+1\right)}\right)^{\ell}\\ &\quad\times e^{-(1+\frac{1}{\ln\rho})\frac{\delta}{\rho\beta_1}}\left(1-e^{-\frac{1}{\rho}(\ln\rho-(1+\frac{1}{\ln\rho})\frac{\delta}{\beta_1})}\right)\end{aligned} \tag{34}$$

whose diversity order can be shown equal to $\ell+1$. As for the second expectation term of $F(\Delta',\ell)$, the same result as (32) is obtained, thus leading to $F(\Delta',\ell) \overset{d}{=} \rho^{-\ell}$. Further, due to the fact that $\Pr\{\Delta < a < \delta\} \doteq \Pr\{\Delta < a < \Delta'\} \doteq \frac{\delta-\Delta}{\rho\beta_1}$, by (33), we can find that if $\lambda < 1$, $F(\Delta,\ell)$ is of the diversity order of ρ^{-1} and for $\ell \geq 2$, $F(\Delta,\ell) \doteq \Pr\{\Delta < a < \delta\} \doteq \frac{\delta-\Delta}{\rho\beta_1}$.

APPENDIX C
PROOFS OF PROPOSITION 3

To characterize $G_1(\Delta,\ell)$ in (13), we first assume without loss of generality that the relays $r_1,\ldots,r_q$ after ARQ0 form the set $\mathcal{Q}$ with $|\mathcal{Q}| = q$. Since there are q different possible active relays in each ARQ, the total number of possible permutations of the active relays in ℓ rounds of ARQs is q^{ℓ}. Let p_i denote the event of choosing the i-th possible permutation of relays from $\mathcal{Q}$, for $i \in \mathcal{I}_1^{q^{\ell}}$. The outage probability of ℓ consecutive ARQ events can thus be expressed as $\sum_{i=1}^{q^{\ell}}\Pr\{p_i\}\Pr\{\mathcal{O}_i\}$ where $\mathcal{O}_i$ denotes the outage events of the ℓ ARQs conditioned on the i-th permutation of relays. Under the assumption of $h_{j,rd}$ having the same variance, $\forall j$, we have $\Pr\{p_i\} = (1/q)^{\ell}, \forall i \in \mathcal{I}_1^{q^{\ell}}$.

Furthermore, we use $X_{r_q}^{n}$ to represent the event of n ARQ rounds through the relay r_q. Given that $\Pr\{\mathcal{O}_i\} = \Pr\{\mathcal{O}_j\}$ if permutation i and j have the same combination of active relays, we apply the Binomial theorem to find all the distinct combinations as follows

$$\begin{aligned}&(X_{r_1}+\cdots+X_{r_q})^{\ell}\\ &\quad=\sum_{\zeta_{q-1}=0}^{\ell}\mathcal{C}_{\zeta_{q-1}}^{\ell}X_{r_q}^{\ell-\zeta_{q-1}}\times(X_{r_1}+\cdots+X_{r_{q-1}})^{\zeta_{q-1}}\end{aligned} \tag{35}$$

$$\begin{aligned}&=\sum_{\zeta_{q-1}=0}^{\ell}\mathcal{C}_{\zeta_{q-1}}^{\ell}X_{r_q}^{\ell-\zeta_{q-1}}\sum_{\zeta_{q-2}=0}^{\zeta_{q-1}}\mathcal{C}_{\zeta_{q-2}}^{\zeta_{q-1}}X_{r_{q-1}}^{\zeta_{q-1}-\zeta_{q-2}}\\ &\quad\times\cdots\times\sum_{\zeta_1=0}^{\zeta_2}\mathcal{C}_{\zeta_1}^{\zeta_2}X_{r_2}^{\zeta_2-\zeta_1}\times X_{r_1}^{\zeta_1}\end{aligned} \tag{36}$$

where the product $X_{r_q}^{\ell-\zeta_{q-1}}X_{r_{q-1}}^{\zeta_{q-1}-\zeta_{q-2}}\times\cdots\times X_{r_1}^{\zeta_1}$ in (36) shows one type of a combination, and its coefficient is the total number of the permutations that belongs to this combination. The outage probability of $X_{r_q}^{\ell-\zeta_{q-1}}$ in (35) is given by

$$\begin{aligned}&\Pr\left\{a>\Delta,\ \bigcap_{i=1}^{\ell-\zeta_{q-1}}\left(\frac{ab_i^{[q]}}{a+b_i^{[q]}+1}<\delta\right)\right\}\\ &\overset{(a)}{\equiv}\Pr\left\{a>\Delta,\ \bigcap_{i=1}^{\ell-\zeta_{q-1}}\left(\max_{j\in\mathcal{I}_1^q}\frac{ab_{i,j}}{a+b_{i,j}+1}<\delta\right)\right\}\\ &=F(\Delta,q\times(\ell-\zeta_{q-1}))\end{aligned} \tag{37}$$

where $b_i^{[q]}$ stands for the highest $\rho|h_{j,rd}|^2$ in $\mathcal{Q}$ in ARQi with $|\mathcal{Q}| = q$, and $b_{i,j}$ denotes the channel gain $\rho|h_{j,rd}|^2$ in ARQi. The equality (a) holds since given $a > 0$, $\frac{ab}{a+b+1}$ is monotonically increasing w.r.t. $b > 0$. For ARQ rounds through different relays in (36), their outage events are independent, and the outage probabilities of other events $X_{r_{q-1}}^{\zeta_{q-1}-\zeta_{q-2}},\ldots,X_{r_2}^{\zeta_2-\zeta_1}$ and $X_{r_1}^{\zeta_1}$ follow the same form of (37).

Define $\mathcal{F}^{(i)}(\Delta,\zeta,q)$ to be the sum outage probability of ζ retransmissions over all possible permutations of ζ forwarding relays chosen independently each time from the set of $r_1,\ldots,r_i$ in $\mathcal{Q}$ with $|\mathcal{Q}| = q$, namely, the outage probability of the partial event $(X_{r_1}+\cdots+X_{r_i})^{\zeta}$ in (35). In other words, $\mathcal{F}^{(q)}(\Delta,\ell,q) = \sum_{i=1}^{q^{\ell}}\Pr\{\mathcal{O}_i\}$. Using (35)∼(37), for $q \geq 2$, we can express $\mathcal{F}^{(q)}(\Delta,\ell,q)$ by a recursive form of

$$\begin{aligned}\mathcal{F}^{(i)}(\Delta,\zeta_i,q)&=\sum_{\zeta_{i-1}=0}^{\zeta_i}\mathcal{C}_{\zeta_{i-1}}^{\zeta_i}(e^{-\frac{\Delta}{\rho\beta_1}})^{\mu(\zeta_i,\zeta_{i-1})}\\ &\quad\times F(\Delta,q\times(\zeta_i-\zeta_{i-1}))\mathcal{F}^{(i-1)}(\Delta,\zeta_{i-1},q)\end{aligned} \tag{38}$$

with $\mathcal{F}^{(2)}(\Delta,\zeta_2,q) \triangleq \sum_{\zeta_1=0}^{\zeta_2}\mathcal{C}_{\zeta_1}^{\zeta_2}(e^{-\frac{\Delta}{\rho\beta_1}})^{\mu(\zeta_2,\zeta_1)}F(\Delta,q\times(\zeta_2-\zeta_1))\times F(\Delta,q\times\zeta_1)$, where we have $\zeta_q \triangleq \ell$, and the term $e^{-\frac{\Delta}{\rho\beta_1}}$, i.e. $\Pr\{a>\Delta\}$, is applied for a case of a relay in $\mathcal{Q}$ being not selected during the entire ARQs, and we thus define the function $\mu(\zeta_i,\zeta_{i-1}) \triangleq \delta_f[\zeta_i-\zeta_{i-1}]+\delta_f[\zeta_{i-1}]-\delta_f[\zeta_i+\zeta_{i-1}]$ such that if $\zeta_{i-1} = \zeta_i$ or $\zeta_{i-1} = 0$, then $\mu(\zeta_i,\zeta_{i-1}) = 1$; otherwise, $\mu(\zeta_i,\zeta_{i-1}) = 0$. As for $q = 1$, $\mathcal{F}^{(1)}(\Delta,\ell,1)$ is defined as $F(\Delta,\ell)$.

Based on the result of $\Pr\{p_i\} = (1/q)^\ell$ and (38), $G_1(\Delta, \ell)$ can finally be expressed as

$$G_1(\Delta,\ell) = \sum_{q=1}^{m} \mathcal{C}_{m-q}^{m} \left(\Pr\{a \le \Delta\}\right)^{m-q} (\frac{1}{q})^{\ell} \mathcal{F}^{(q)}(\Delta,\ell,q). \tag{39}$$

Appendix D
Proof of Corollary 1

Comparing (13) to (16), in this proof, we only need to show that $\mathcal{F}^{(q)}(\Delta, \ell, q)$ for $\ell > 0$ and $q > 0$ can be summarized as $\widetilde{\mathcal{F}}^{(q)}(\Delta,\ell,q) \triangleq q^\ell \Pr\{a > \Delta\}^q \Pr\{b < \delta\}^{q\times\ell}$ after the form of $F(\Delta, n)$ in (15) are replaced by that of $\widetilde{F}(\Delta, n)$ in (8).

Using the delta function $\delta_f[\cdot]$, we first rewrite $\widetilde{F}(\Delta, n)$ as

$$\widetilde{F}(\Delta,n) \triangleq \Pr\{a > \Delta\}^{1-\delta_f[n]} \Pr\{b < \delta\}^n, \text{ for } n \ge 0. \tag{40}$$

Then, substituting (40) back into (15) or (38), we can obtain $\widetilde{\mathcal{F}}^{(q)}(\Delta, \zeta_q, q)$ which is given by

$$\widetilde{\mathcal{F}}^{(q)}(\Delta,\zeta_q,q) = \sum_{\zeta_{q-1}=0}^{\zeta_q} \mathcal{C}_{\zeta_{q-1}}^{\zeta_q} \Pr\{a > \Delta\}^{\mu(\zeta_q,\zeta_{q-1})+1-\delta_f[q\times(\zeta_q-\zeta_{q-1})]} \times \Pr\{b<\delta\}^{q\times(\zeta_q-\zeta_{q-1})} \times \widetilde{\mathcal{F}}^{(q-1)}(\Delta,\zeta_{q-1},q). \tag{41}$$

By expanding the recursive form of (41) with $\zeta_q = \ell > 0$ and $q > 2$, we have

$$\widetilde{\mathcal{F}}^{(q)}(\Delta,\ell,q) = \sum_{\zeta_{q-1}=0}^{\ell} \sum_{\zeta_{q-2}=0}^{\zeta_{q-1}} \times\cdots\times \sum_{\zeta_1=0}^{\zeta_2} \mathcal{C}_{\zeta_{q-1}}^{\ell} \mathcal{C}_{\zeta_{q-2}}^{\zeta_{q-1}} \times\cdots\times \mathcal{C}_{\zeta_1}^{\zeta_2} \Pr\{a>\Delta\}^{t_q} \Pr\{b<\delta\}^{q\times\ell} \tag{42}$$

where $t_q \triangleq \mu(\ell, \zeta_{q-1}) + 1 - \delta_f[q\times(\ell-\zeta_{q-1})] + \sum_{j=q-1,\ldots,2} \{\mu(\zeta_j, \zeta_{j-1}) + 1 - \delta_f[q \times (\zeta_j - \zeta_{j-1})]\} + 1 - \delta_f[q \times \zeta_1]$, and $\ell \ge \zeta_{q-1} \ge \cdots \ge \zeta_1 \ge 0$.

By the definition of $\mu(\zeta_i, \zeta_{i-1})$, the exponent t_q can be further simplified as

$$\begin{aligned} t_q &\overset{(a)}{=} \delta_f[\zeta_{q-1}] + 1 - \delta_f[\zeta_q+\zeta_{q-1}] + \sum_{j=q-1,\ldots,2} \{\delta_f[\zeta_{j-1}] + 1 - \delta_f[\zeta_j+\zeta_{j-1}]\} + 1 - \delta_f[\zeta_1] \\ &\overset{(b)}{=} -\delta_f[\zeta_q+\zeta_{q-1}] + \sum_{j=q,\ldots,3} \{\delta_f[\zeta_{j-1}] + 1 - \delta_f[\zeta_{j-1}+\zeta_{j-2}]\} + 1 + 1 \\ &\overset{(c)}{=} 0 + (q-2) + 2 \end{aligned} \tag{43}$$

where the equality (a) is based on the definition of $\mu(\zeta_i, \zeta_{i-1})$ and the fact that $\delta_f[q(\zeta_j - \zeta_{j-1})] = \delta_f[\zeta_j - \zeta_{j-1}]$ for $q > 0$, and (b) is just to reduce the expression and rearrange its summation index, and finally, (c) results from $\delta_f[\zeta_q + \zeta_{q-1}] = 0$ since $\zeta_q = \ell > 0$, and the fact that $(\delta_f[\zeta_{j-1}] + 1 - \delta_f[\zeta_{j-1} + \zeta_{j-2}])$ is equal to 1 due to $\zeta_{j-1} \ge \zeta_{j-2} \ge 0$.

As a result, for $\ell > 0$ and $q > 2$, we arrive at

$$\begin{aligned} \widetilde{\mathcal{F}}^{(q)}(\Delta,\ell,q) &= \Pr\{a>\Delta\}^q \Pr\{b<\delta\}^{q\ell} \times \sum_{\zeta_{q-1}=0}^{\ell} \sum_{\zeta_{q-2}=0}^{\zeta_{q-1}} \times\cdots\times \sum_{\zeta_1=0}^{\zeta_2} \mathcal{C}_{\zeta_{q-1}}^{\ell} \mathcal{C}_{\zeta_{q-2}}^{\zeta_{q-1}} \times\cdots\times \mathcal{C}_{\zeta_1}^{\zeta_2} \\ &= \Pr\{a>\Delta\}^q \Pr\{b<\delta\}^{q\ell} \times q^\ell. \end{aligned} \tag{44}$$

As for the cases of $\widetilde{\mathcal{F}}^{(2)}(\Delta, \ell, 2)$ and $\widetilde{\mathcal{F}}^{(1)}(\Delta, \ell, 1)$, by similar steps, we can derive the same result of (44) for $q \in \mathcal{I}_1^2$ from the original equation (15). This ends the proof.

Appendix E
The Diversity Analysis for $\mathcal{P}_{\text{OUT},n}^{\text{SOAF-A}}$ With $\Delta := \lambda\delta < \delta$

Given $\Delta := \lambda\delta < \delta$, according to Lemma 2, we can rewrite $F(\Delta,\ell) \overset{d}{=} (1/\rho)^{1-\delta_f[\ell]}$ for $\ell \ge 0$. With this result, the recursive formula of $\mathcal{F}^{(q)}(\Delta, \zeta_q, q)$ with $\zeta_q = \ell > 0$ in (15) or (38) can be expanded from a diversity analysis viewpoint as follows:

$$\begin{aligned} \mathcal{F}^{(q)}(\Delta,\zeta_q,q) &\overset{d}{=} \sum_{\zeta_{q-1}=0}^{\zeta_q} \left(\frac{1}{\rho}\right)^{1-\delta_f[q\times(\zeta_q-\zeta_{q-1})]} \times \mathcal{F}^{(q-1)}(\Delta,\zeta_{q-1},q) \\ &\overset{d}{=} \sum_{\zeta_{q-1}=0}^{\zeta_q} \sum_{\zeta_{q-2}=0}^{\zeta_{q-1}} \times\cdots\times \sum_{\zeta_1=0}^{\zeta_2} \left(\frac{1}{\rho}\right)^{t'_q}, \text{ for } q>2, \end{aligned} \tag{45}$$

where $t'_q \triangleq \sum_{j=q,\ldots,2} (1 - \delta_f[q \times (\zeta_j - \zeta_{j-1})]) + 1 - \delta_f[q \times \zeta_1]$ and $\zeta_q, \ldots, \zeta_1$, are integers that satisfy $\zeta_q = \ell > 0$ and $\zeta_q \ge \zeta_{q-1} \ge \cdots \ge \zeta_1 \ge 0$.

We next show the smallest value of t'_q is equal to 1. Due to $q > 0$, t'_q can be reduced as

$$t'_q = 1 - \delta_f[\zeta_q - \zeta_{q-1}] + \sum_{j=q-1,\ldots,2} (1 - \delta_f[\zeta_j - \zeta_{j-1}]) + 1 - \delta_f[\zeta_1]. \tag{46}$$

Apparently, t'_q is a non-negative integer, and the equality for $t'_q = 0$ occurs only if $\zeta_1 = \zeta_2 = \cdots = \zeta_q = 0$. The condition of $\zeta_q = 0$ doesn't satisfy that $\zeta_q = \ell > 0$. In other words, the smallest value of t'_q will be equal to 1, which occurs if $\zeta_1 = \zeta_2 = \cdots = \zeta_{q-1} = 0$.

As a result, $\mathcal{F}^{(q)}(\Delta, \ell, q) \overset{d}{=} 1/\rho$ for $q > 2$. As for the cases of $\mathcal{F}^{(1)}(\Delta, \ell, 1)$ and $\mathcal{F}^{(2)}(\Delta, \ell, 2)$, by similar steps, we have the same result of $\mathcal{F}^{(q)}(\Delta, \ell, q) \overset{d}{=} 1/\rho$ for $q \in \mathcal{I}_1^2$. Thus, we know by (39) that $G_1(\Delta, \ell) \overset{d}{=} 1/\rho$ for $\lambda < 1$, $\ell > 0$. Substituting the result back into (13) yields

$$\mathcal{P}_{\text{out},n}^{\text{SOAF-A}} \overset{d}{=} \frac{1}{\rho} \times \sum_{\ell=0}^{n} \left(\frac{1}{\rho^{m+1}}\right)^{n-\ell} \times \left(\frac{1}{\rho}\right)^{1-\delta_f[\ell]} \overset{d}{=} \frac{1}{\rho} \times \frac{1}{\rho} \tag{47}$$

whose diversity order is equal to two. This ends the proof.

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